

From “Dark Stars” to Gravitational Collapse within Einstein’s Theory. The Emergence of Relativistic Astrophysics

Luisa Bonolis

A Century of General Relativity
2-5 December
Berlin

Study of Super-Dense Matter

Quantum Mechanics

Astrophysics

General Relativity

Cosmology

Astronomy

Nuclear Physics

Condensed Matter
Physics

Particle Physics

Stellar
evolution

Stellar
structure

Nuclear Astrophysics

Origin of cosmic rays

Stellar
energy

Origin of light
and heavy chemical
elements

Stars as Physics Laboratories

Astrophysics

1934
Hypothesis of
Neutron Stars
???

Neutron
Stars

Cosmology

Radio Astronomy

General Relativity

Nuclear Astrophysics

Cosmic Physics

Discovery of the neutron

Nuclear Physics

Emergence of
the concept of
compact stars

Quantum Mechanics and Special Relativity

Neutron
cores

Collapsing
Neutron
cores

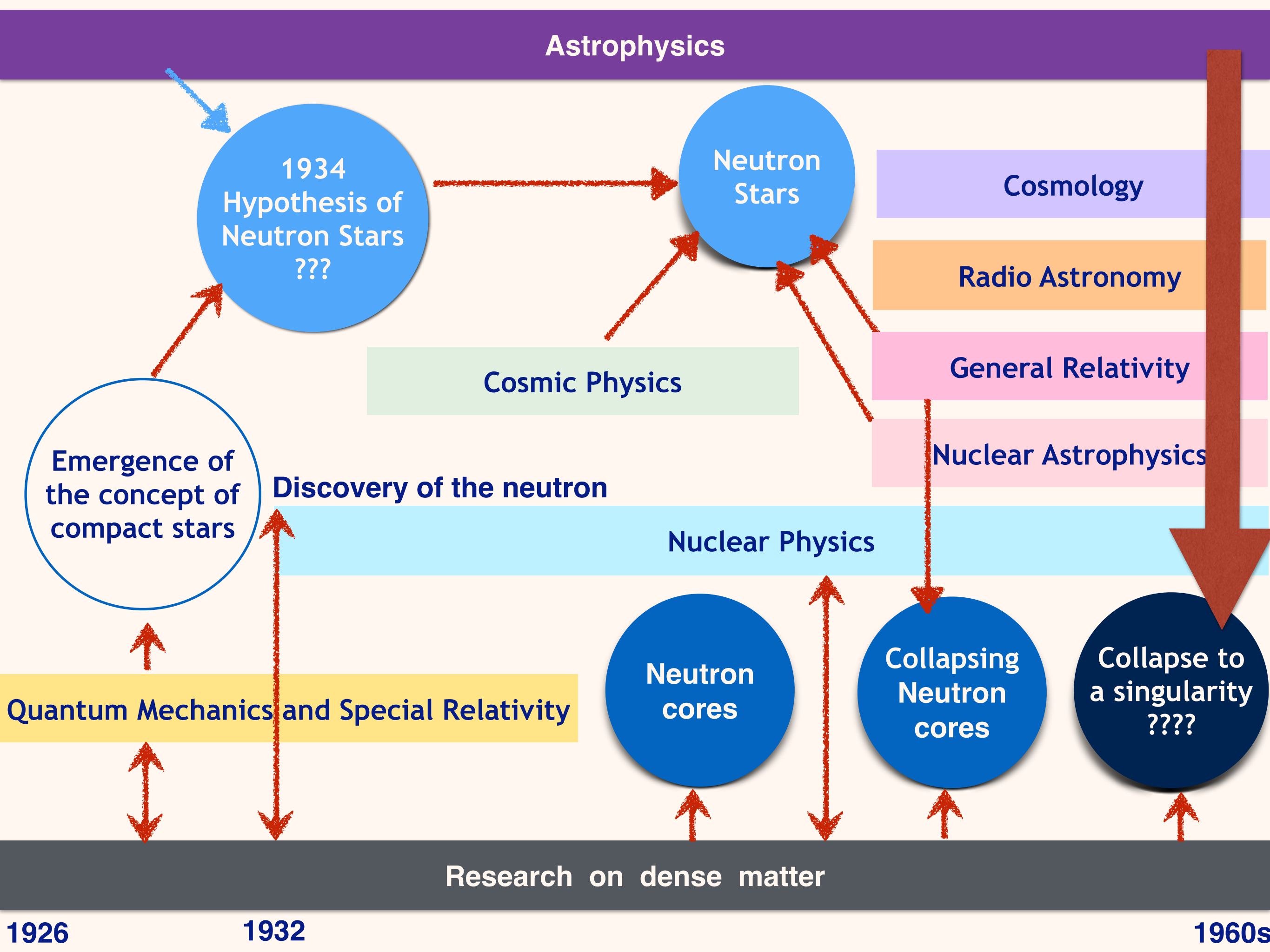
Collapse to
a singularity
????

Research on dense matter

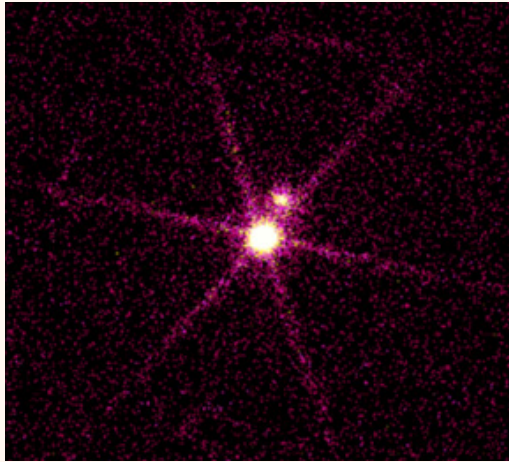
1926

1932

1960s



Emergence of the concept of compact stars



Eddington (1922): “Strange objects which persist in showing a type of spectrum entirely out of keeping with their luminosity, *may ultimately teach us more than a host which radiate according to rule.*”

White dwarfs
as physics
laboratories



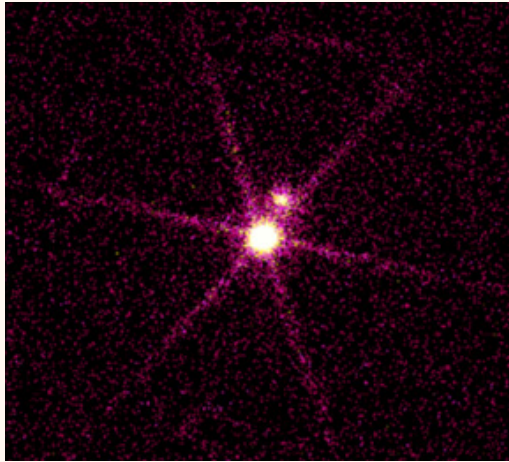
Measurements of redshift were made by Adams and published in 1925. But such densities, unparalleled in terrestrial experience, appeared to be *absurd*

Eddington: “Prof. Adams has killed two birds with one stone; he has carried out a new test of Einstein's theory of general relativity and he has confirmed our suspicion that matter 2000 times denser than platinum is not only possible, but is actually present in the universe”

First connection between a compact star and general relativity

Research on dense matter

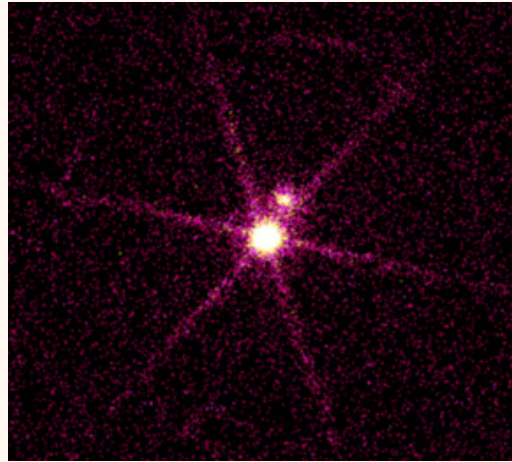
Emergence of the concept of compact stars



Eddington, *Internal Constitution of Stars*
(1926): "It would seem that the star will be in an awkward predicament when its supply of subatomic energy ultimately fails...How does a white dwarf manage to cool down?"

White dwarfs
as physics
laboratories

Emergence of the concept of compact stars



Fowler, *Statistical Mechanics*: "The absolutely final state is one in which there is only one possible configuration left. We may perhaps venture to refer to their probable final state as the black dwarf stages... The black dwarf material is best likened to a single gigantic molecule in its lowest quantum state."

White dwarfs
as physics
laboratories

Dense degenerate gas of
electrons and bare nuclei

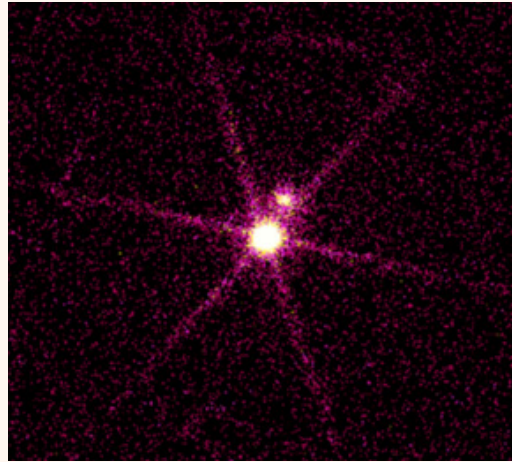
Quantum Mechanics

Fowler

Research on dense matter

1926

Emergence of the concept of compact stars



White dwarfs
as physics
laboratories

1926 Fowler *On Dense Matter*
Dense degenerate gas of
electrons and bare nuclei.
Analogy with gigantic molecule
in its lowest quantum state

Chandrasekhar Limit

Maximum mass for a
stable white dwarf star.

Quantum Mechanics

Quantum Mechanics and Special Relativity

Fowler

Anderson,
Stoner,
Frenkel

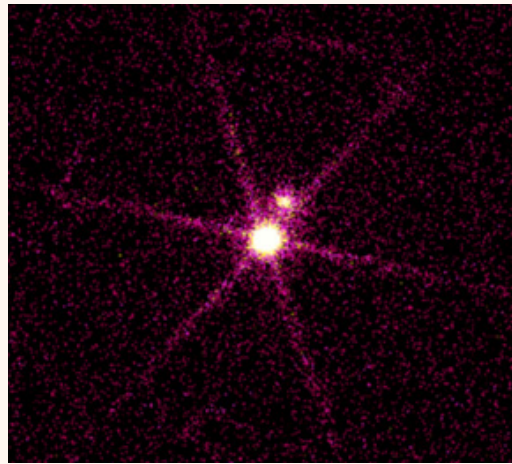
Chandrasekhar

Research on dense matter

1926

1929/1931

Emergence of the concept of compact stars



White dwarfs
as physics
laboratories

A new state of matter

1932: “Life histories of stars of small and large mass are radically different.”

1934: “A star of mass larger than the critical one cannot pass into the white-dwarf stage and one is left speculating on other possibilities...”

1926 Fowler *On Dense Matter*
Dense degenerate gas of electrons and bare nuclei.
Analogy with gigantic molecule in its lowest quantum state

Chandrasekhar Limit

Maximum mass for a stable white dwarf star.

Quantum Mechanics

Quantum Mechanics and Special Relativity

Fowler

Anderson,
Stoner,
Frenkel

Chandrasekhar

Chandrasekhar

Research on dense matter

1926

1929/1931

1934-1935

Dense matter in Cosmology and Astrophysics

1931

The “primaeval
atom” of
Georges Lemaître

Protons, electrons and
alpha-particles packed
together at nuclear
densities

A Friedman universe
evolving from a condensed
material pre-universe



June 1931. Langer and Rosen. *The Neutron*

Postulating the existence of the neutron, a
combination of an electron and a proton, useful
in explaining high density matter in stars.

Disintegration of the huge
unstable super atom

Origin of
chemical elements

Origin of high energies
of cosmic rays

The neutron

Nuclear Physics

Research on dense matter

1931

1932

Interlude: Landau and the prototype of a new physical system: the *neutron core*

1931/1932 - Landau
On the Theory of Stars
Stellar core of dense
nuclear matter (still
protons and electrons)

First theoretical
analysis of the stability
of a dense core in stars
as a physical system

Atomic nuclei in close
contact “forming one
gigantic nucleus
themselves”

“There exist in the whole quantum theory no cause preventing the system from collapsing to a point : As in reality such masses exist quietly as stars and do not show any such ridiculous tendencies we must conclude that all stars heavier than 1.5 solar masses certain possess regions in which the laws of quantum mechanics (and therefore of quantum statistics) are violated...”



The neutron

Nuclear Physics

Research on dense matter

1931

1932

Dense matter entering the nuclear age

Interior of stars transformed into a dense gas of neutrons

Model for theorizing on nuclear matter, the new frontier of microscopic physics

1931/1932 -
Landau
Stellar core of
dense nuclear
matter

1933 - Sterne
Compression of dense
matter (neutrons,
protons, electrons and
nuclei) inducing
formation of neutrons

The neutron

Nuclear Physics

Research on dense matter

1931

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1933

Dense matter entering the nuclear age

Interior of stars transformed into a dense gas of neutrons

Model for theorizing on nuclear matter, the new frontier of microscopic physics

Isolated speculation

1934 - Baade & Zwicky

- Very small radius
- Very high density
- Extremely closely packed neutrons

Origin of cosmic rays

Invention of the concept of Neutron Star as remnant of supernova explosion

1931/1932 - Landau
Stellar core of dense nuclear matter

1933 - Sterne
Compression of dense matter (neutrons, protons, electrons and nuclei) inducing formation of neutrons

The neutron

Nuclear Physics

Research on dense matter

1931

1932

1933

1934

Gamow, the king of borderline problems



1936 - Gamow

- Stellar core as a dense gas of neutrons
- A gas of neutrons could be compressed to a much higher density than a gas of nuclei and electrons analogous to the conditions inside an atomic nucleus
- Probable densities of such stars: about 10^{17} kg/m^3

1935 - Gamow
Neutronic stellar
nucleus playing a
role in the formation
of elements in stars

Nuclear Physics

Research on dense matter

1935

1936/1937

The neutron core in action



1936 - Gamow

- Stellar core as a dense gas of neutrons
- A gas of neutrons could be compressed to a much higher density than a gas of nuclei and electrons analogous to the conditions inside an atomic nucleus
- Probable densities of such stars: about 10^{17} kg/m^3

1935 - Gamow
Neutronic stellar nucleus playing a role in the formation of elements in stars

1936 - Hund
Matter at extremely high Pressures and Temperatures

1937/1938 - Kothari
Formation of neutrons inside white dwarfs

Nuclear Physics

Research on dense matter

1935

1936/1937

1936

1938

Dense cores and stellar energy

1938 - Landau - *Stellar Energy*
Maximum mass of a neutron
star and gravitational collapse
of
super-dense neutron core as
source of stellar energy



Stellar energy

Nuclear Physics

Research on dense matter

1938

Dense neutron cores and stellar energy

1938 - Landau - *Stellar energy*
Maximum mass of a neutron
star and gravitational collapse
of
super-dense neutron core as
source of stellar energy



March 1938 - Gamow, Teller and Tuve organize the
4th Washington Conference on Theoretical Physics

- Problems connected to stellar energy and nuclear physics
- Discussions on the possible existence of a super-dense stellar nucleus as proposed by Landau



1938



1938

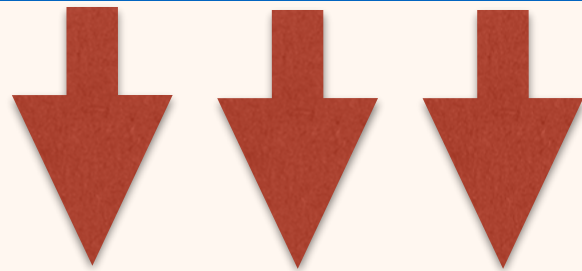
Stellar energy

Nuclear Physics

Research on dense matter

From stellar energy to stability of neutron cores

Gamow 1936 and Landau 1938
suggestion of a condensed and
collapsing neutron core as
source of stellar energy



1938 Oppenheimer & Serber
On the stability of Stellar Neutron Cores

“Essential for a discussion of the role of
such a core, is the estimate of the
minimum mass for which it will be stable.”



1939 Bethe
Energy production in stars

Stellar energy

Nuclear Physics

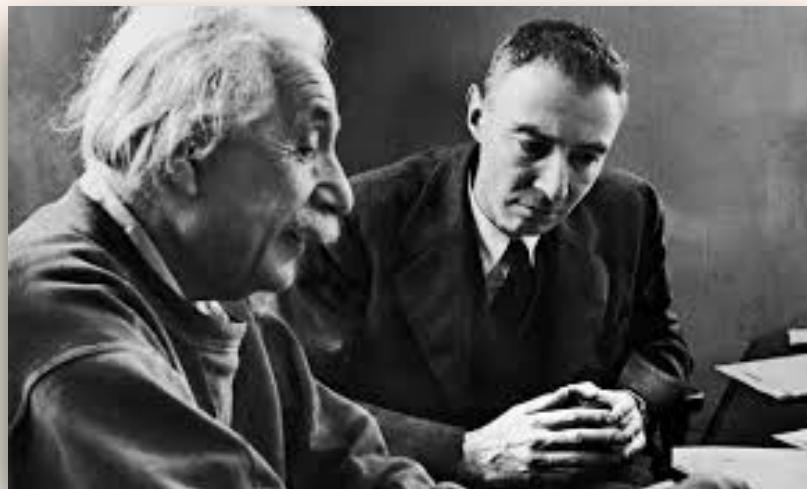
Research on dense matter

1938

1939

From stellar energy to gravitational collapse

February 1939 - Richard Tolman
*Static solutions of Einstein's Field
equations for spheres of fluid*



February 1939
Oppenheimer & Volkoff
On Massive Neutron Cores

- General Relativity and
numerical analysis of
stability of massive neutron
core in stars

General Relativity

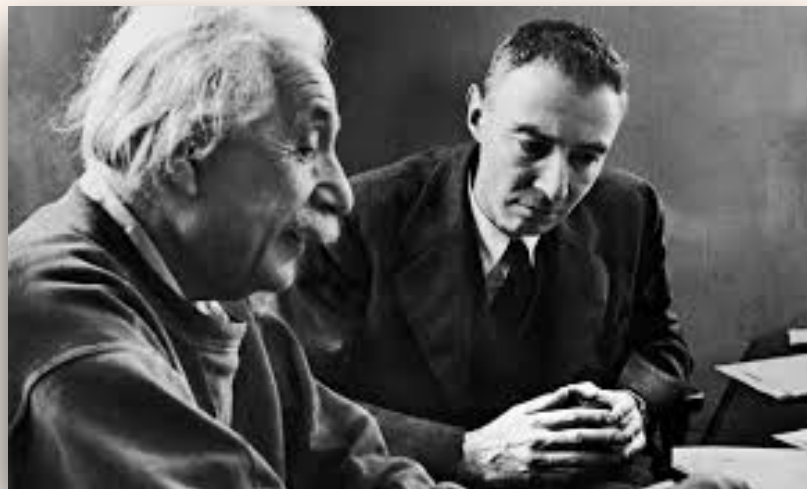
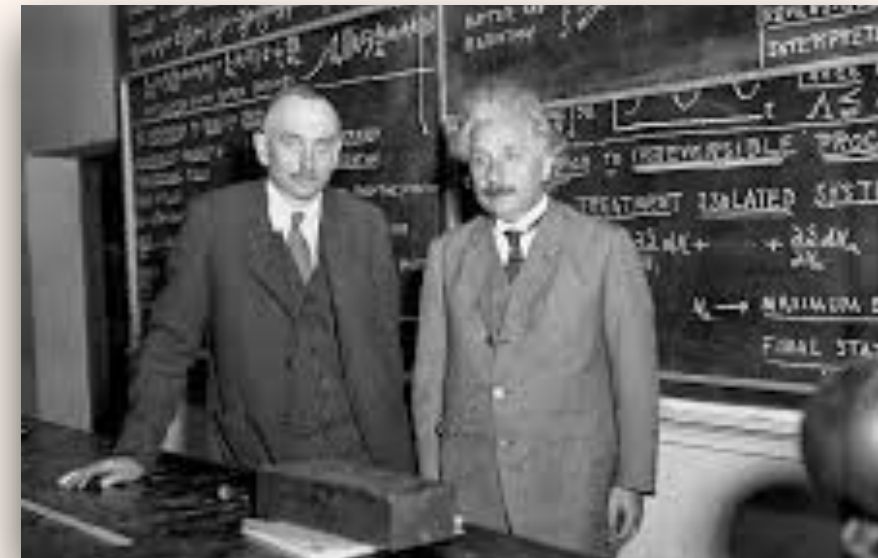
Nuclear Physics

Research on dense matter

1939

From stellar energy to gravitational collapse

February 1939 - Richard Tolman
*Static solutions of Einstein's Field
equations for spheres of fluid*



February 1939
Oppenheimer & Volkoff
On Massive Neutron Cores

- General Relativity and numerical analysis of stability of massive neutron core in stars

September 1939
Oppenheimer & Snyder
*On Continued Gravitational
Contraction*

General Relativity

Nuclear Physics

Research on dense matter

1939

1939

Oppenheimer and Snyder are not alone...

Discussions with Tolman



1938 - Zwicky

On collapsed neutron stars

- General Relativity
- Theory of critical mass

1939 - Zwicky

On the Theory and Observation of Highly Collapsed Stars

May 1938 Datt
On a Class of Solutions of the Gravitation Equations of Relativity

General Relativity

Nuclear Physics

Research on dense matter

1938

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Oppenheimer and Snyder are not alone...

Discussions with Tolman



1938 - Zwicky
On collapsed neutron stars
• General Relativity
• Theory of critical mass

1939 - Zwicky
On the Theory and Observation of Highly Collapsed Stars

May 1938 Datt
On a Class of Solutions of the Gravitation Equations of Relativity

July 1939 Cernuschi
Super-Novae and the Neutron-Core Stars
Physical basis to Zwicky's idea

General Relativity

Nuclear Physics

Research on dense matter

1938

1938

1939

Oppenheimer and Snyder are not alone...

Discussions with Tolman



1938 - Zwicky
On collapsed neutron stars

- General Relativity
- Theory of critical mass

1939 - Zwicky
On the Theory and Observation of Highly Collapsed Stars

Zwicky: "It is impossible to observe physical conditions in stellar bodies which have reached the Schwarzschild limit..."

May 1938 Datt
On a Class of Solutions of the Gravitation Equations of Relativity

July 1939 Cernuschi
Super-Novae and the Neutron-Core Stars
Physical basis to Zwicky's idea

October 1939 Einstein
On a Stationary System With Spherical Symmetry Consisting of Many Gravitating Masses

General Relativity

& Astrophysics

Nuclear Physics

Research on dense matter

1938

1938

1939

1939

War and early post-War years

- 1938 - von Weizsäcker, *Über Elementumwandlungen im Innern der Sterne*. The density of the early hot universe is of the same order as the density of an atomic nucleus
- 1940 - Gamow, *The Birth and Death of the Sun*: radioactive elements formed shortly after the creation of the universe from the primordial superdense gas of neutrons and protons
- 1941 - April Gamow and Schönberg. *Neutrino Theory of Stellar Collapse*, cooling due to neutrino losses by the urca-process in inverse beta-decay would result in a catastrophic failure of pressure support near the centre.
- 1942 - Eighth Washington Conference on Theoretical Physics: Stellar Evolution and Cosmology. The problem of nucleosynthesis became a hot and open one. Where they formed in an earlier prestellar stage of the Universe?
- 1944 - Jordan. *Über die Entstehung der Sterne*
- 1946 - Jordan, *Zur Lösung des Paradoxons von Chandrasekhar*
- 1947 - van Albada. *On the Origin of the Heavy Elements*. Compressed neutralized matter in massive stars and formation of elements, explosion and distribution in space
- 1947 - Jordan. *Die Herkunft der Sterne*, mentioning the formation of a neutron cores and the limiting mass
- 1948 - Gamow's group and investigations about connection between nuclear reactions and cosmology. Alpher Bethe Gamow paper on *The origin of chemical elements* drawing attention to the necessity of a hot, dense phase in the early Universe if the light elements were to be synthesized cosmologically
- Neutron capture cross sections became available after the war together with the abundances charts showing some striking regularities that eventually led to the Shell model of the nucleus formulated independently by Goeppert-Mayer and Jensen in Germany.
- End of the 1940s: emergence of the Steady State Theory of Bondy, Gold and Hoyle

Scientists emerging from hydrogen bomb efforts

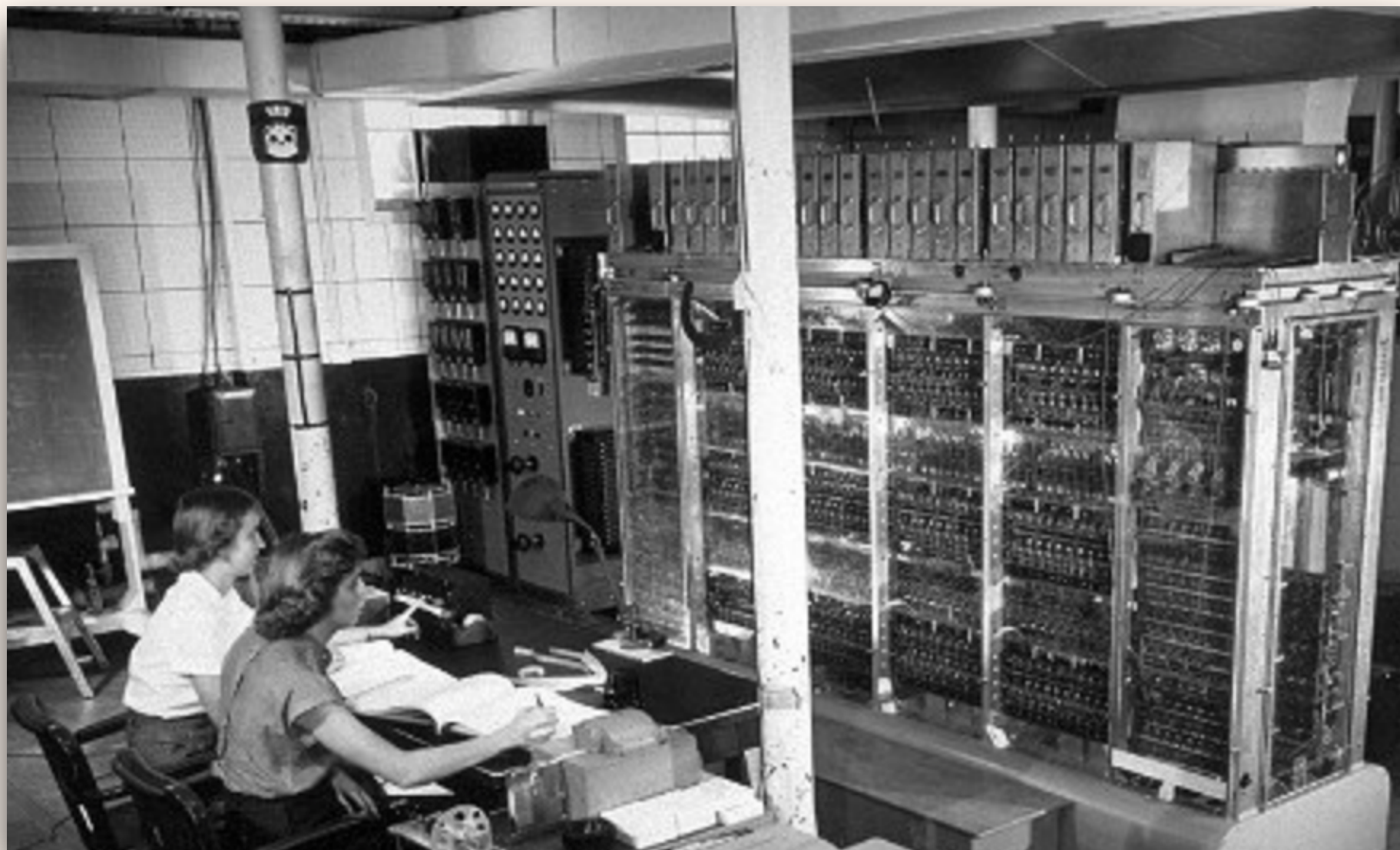
New tools of post-war science

- Advances in nuclear science
- First powerful computers

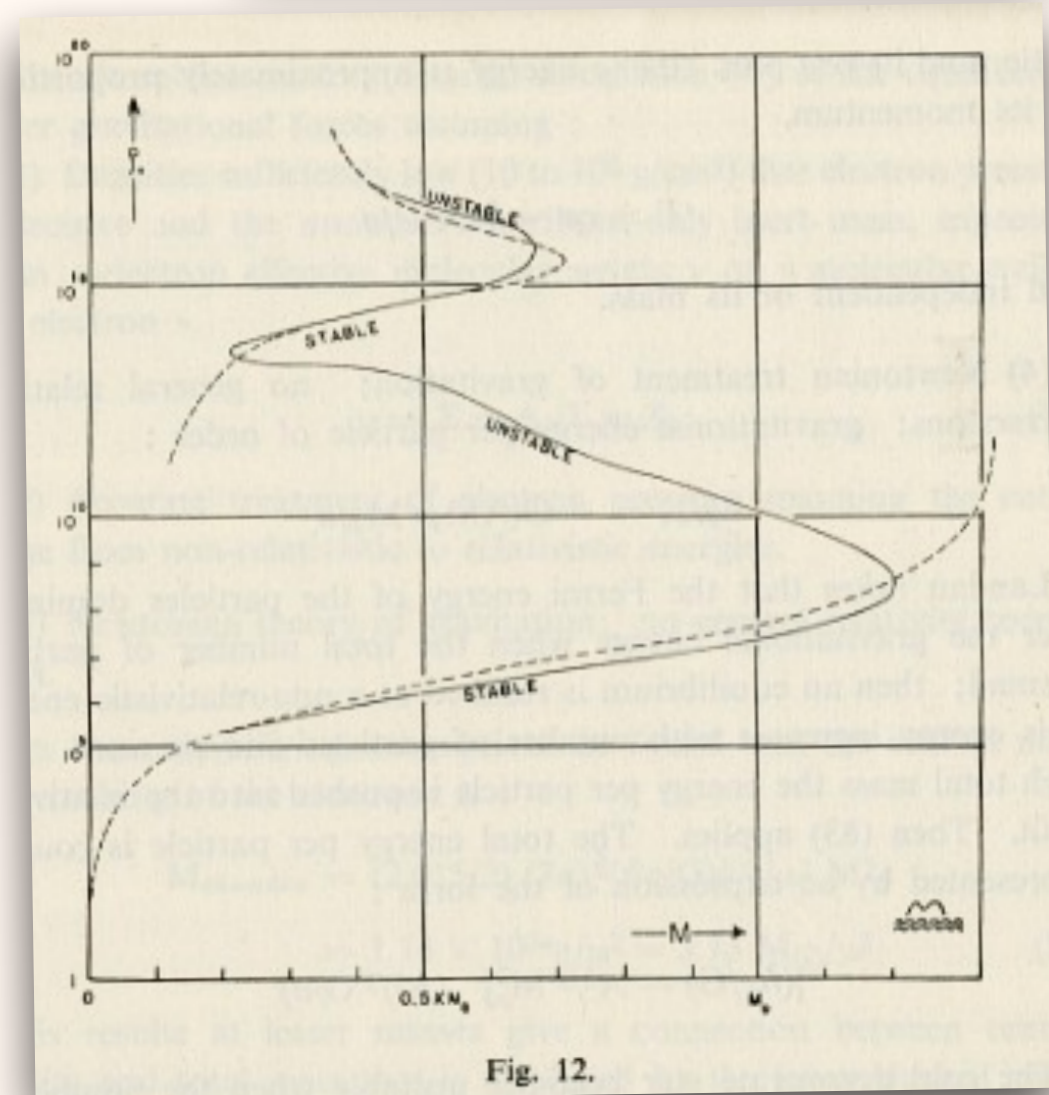
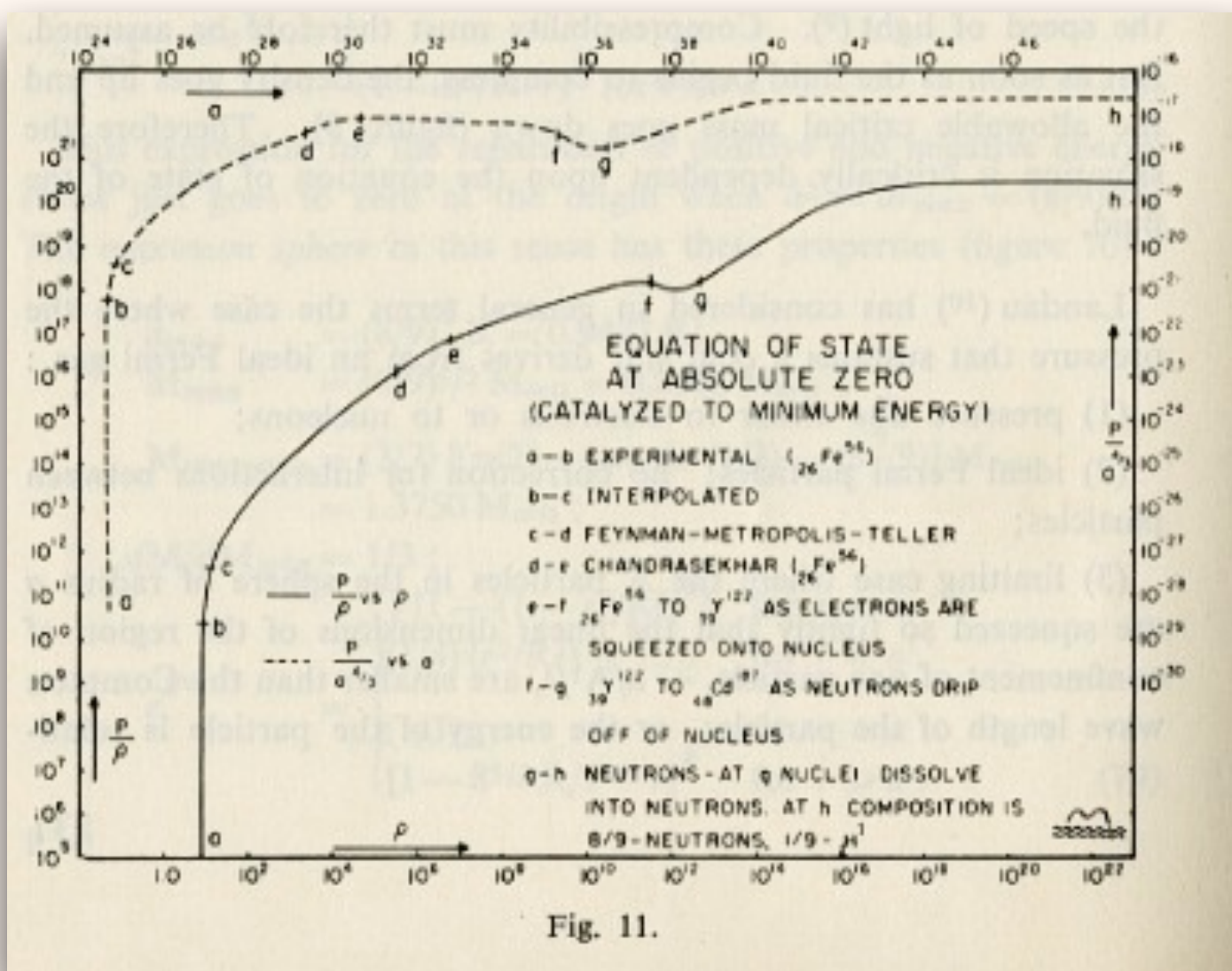
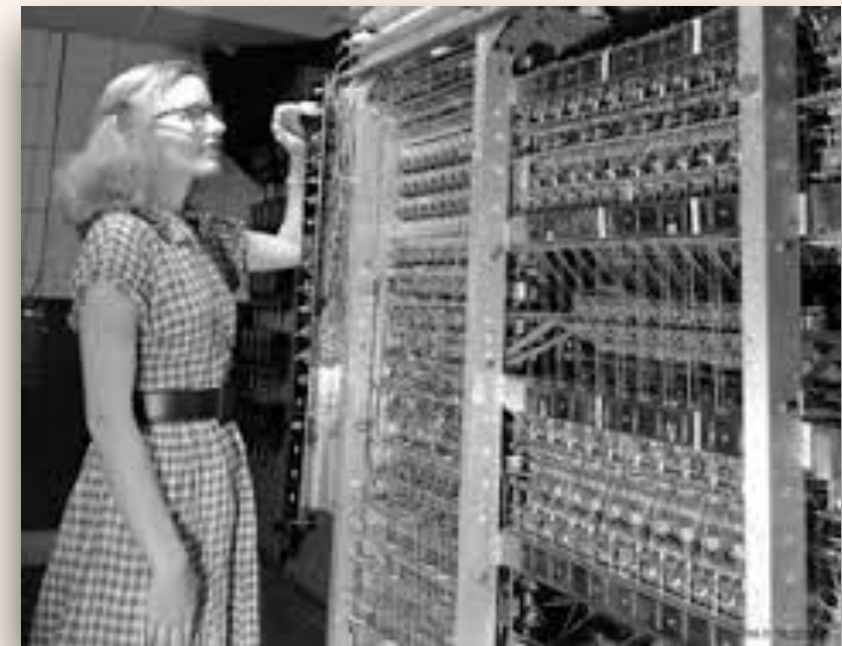
J. A. Wheeler (US) and Y. B. Zel'dovich (USSR)

Having remarkably parallel interests...

- Exploring the potentialities of Einstein's theory
- Formation of schools



Wheeler's fight against the singularity



Structure and evolution of the universe

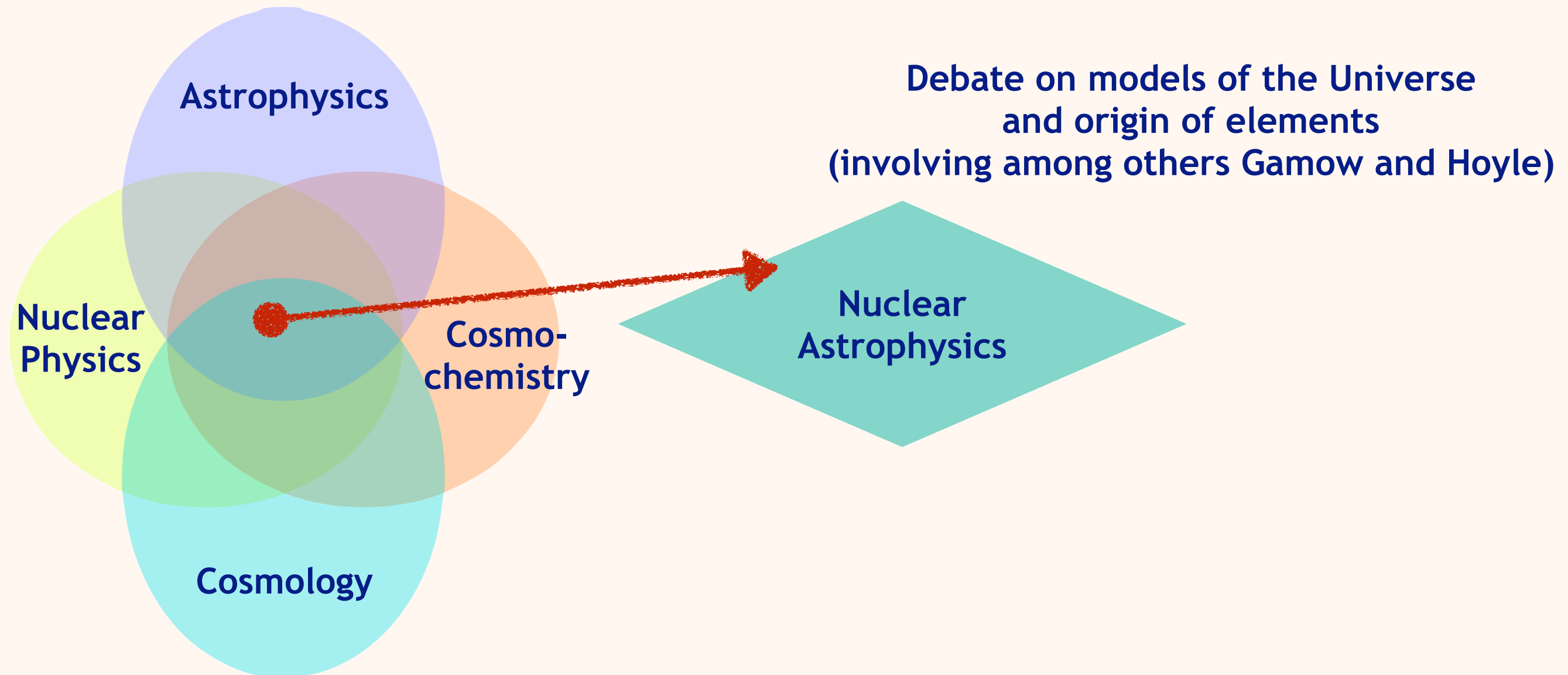
Solvay conference 1958



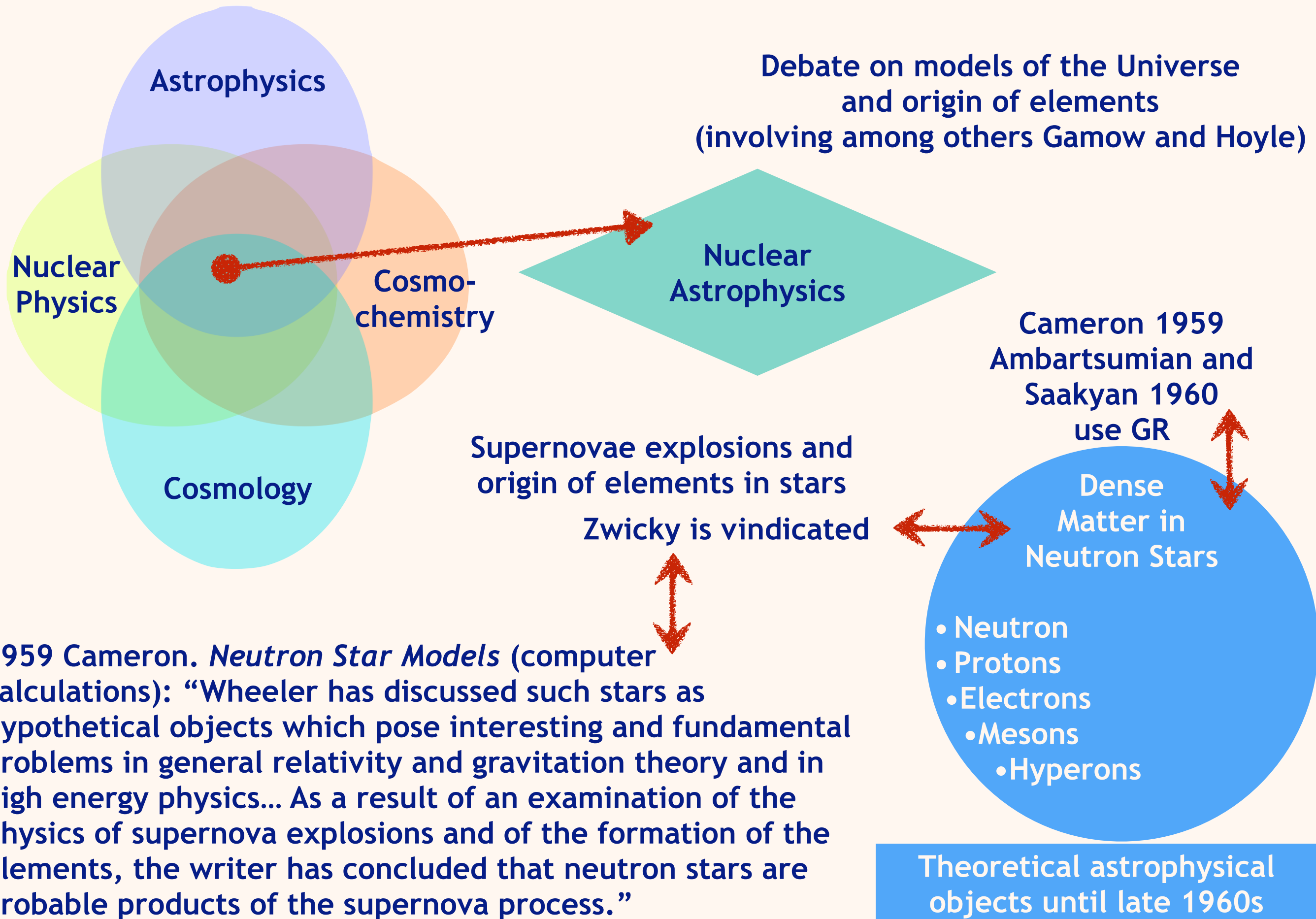
Sitting, from left : McCrea, Oort, Lemaitre, Gorter, Pauli, Bragg, Oppenheimer, Møller, Shapley, Heckmann

Standing, from left: Klein, Morgan, Hoyle, Kukaskin, Ambartzumian, van de Hulst, Fierz, Sandage, Baade, Schatzman, Wheeler, Bondi, Gold, Zanstra, Rosenfeld, Ledoux, Lovell, Geneniau

Relativistic cosmology and nuclear astrophysics



Relativistic cosmology and nuclear astrophysics



Relativistic cosmology and radio astronomy

Post-War
Science

Scientists emerging
from Radar Research
in WWII
Birth of
Radioastronomy





Extragalactic sources
as probes of
cosmological models

Debate on models of the Universe

- “Steady state” (Bondi, Gold and Hoyle): the Universe is the same everywhere and everywhen
- “Big bang” theories (Gamow and collaborators): the Universe was different in the past
- Number of intense sources increasing with distance
- Galaxies producing more radio waves in the past



- 
- Radio catalogues
 - Distribution of radio-sources



Conflict with
steady-state
cosmology

Radio astronomy unveils a violent universe

July-August 1958. International Paris Symposium on Radio Astronomy

Enigma of the nature of powerful radio sources

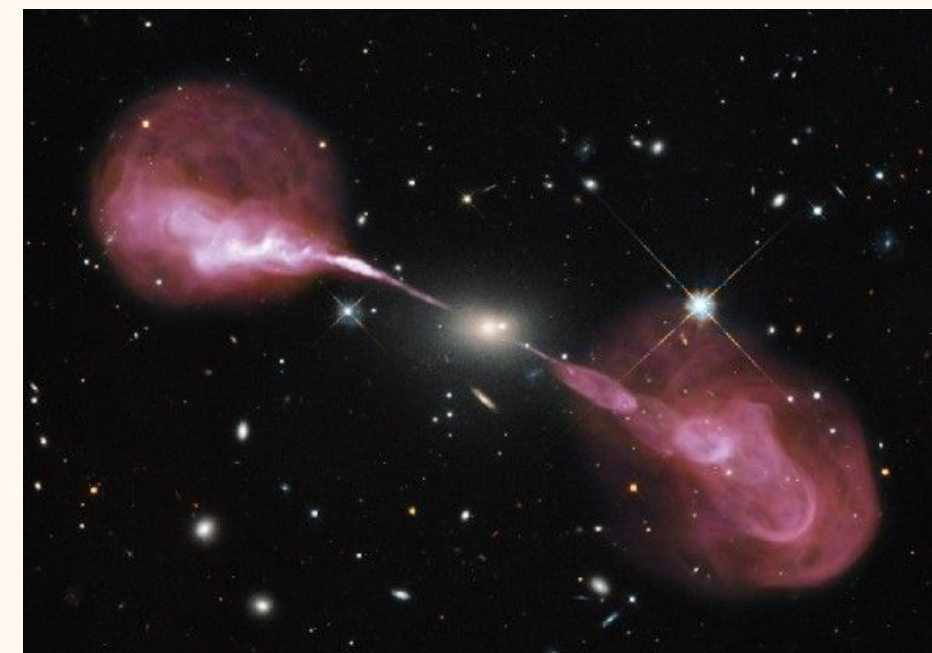


Crab Nebula
remnant of supernova explosion



1959 - G Burbidge. *Estimates of the Total Energy in Particles and Magnetic Field in the Non-Thermal Radio Sources*

the beginning of modern high-energy astrophysics



Jets of particles accelerated to relativistic speed by the magnetic fields around the center of the galaxy Hercules A

Quasi stellar radio sources

October 1961

Ginzburg *On the nature of the radio galaxies*. Suggested that gravitational energy released in contraction might account for the radio sources

February 1963

Hoyle and Fowler suggest that those tremendous energies in distant galaxies might be supplied by the gravitational collapse of a supermassive star (thousands of supernovas)

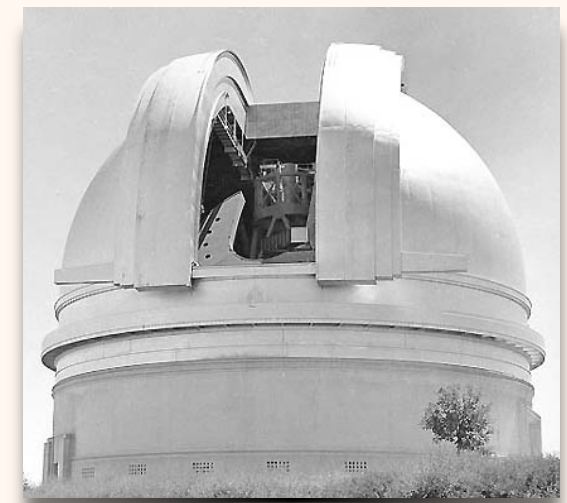
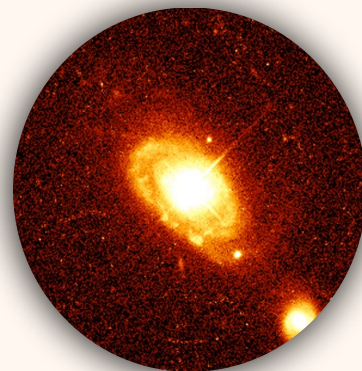
Quasi stellar radio sources

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March 16 1963 issue of *Nature*

Hazard et al. (1963), Schmidt (1963), Oke (1963), and Greenstein and Matthews (1963)
published as consecutive papers

Schmidt 1963: *3C 273: a star-like object with large red-shift.*

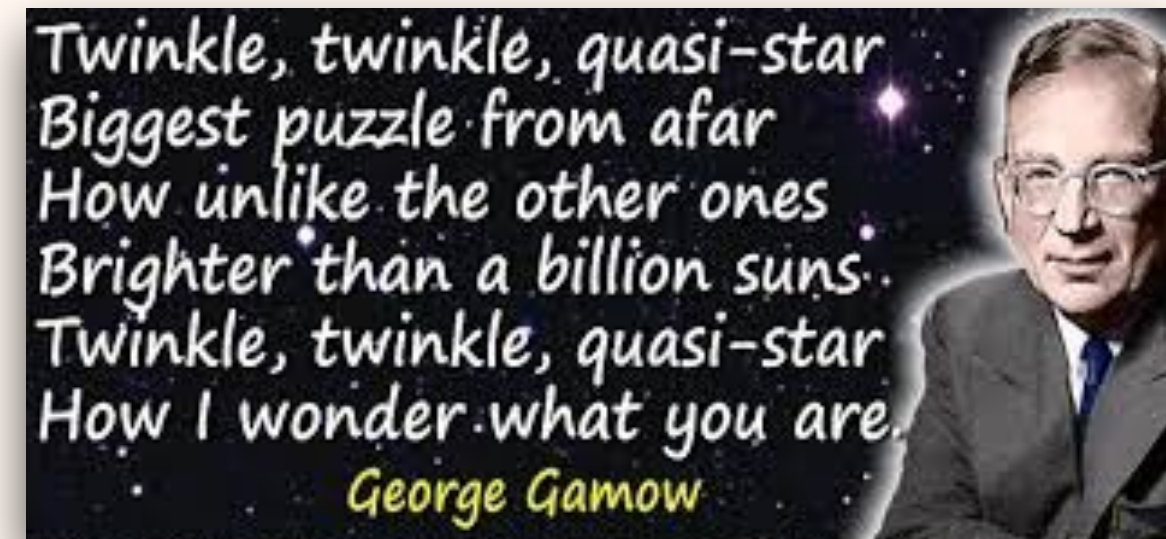
- 3 billion light-years away from Earth
- 4 trillion times brighter than the Sun

Reactions to the Fowler-Hoyle paper

- 1962-1963. Feynman offered a lecture course on GR at Caltech
- 1963 (received May 13) Iben, *Massive Stars in Quasi-Static Equilibrium* (Collapse and general relativity) Thanks Feynman for suggesting the problem and Fowler and Michel for discussions
- 1963 (received May 13) Michel. *Collapse of Massive Stars*. It is the purpose of this note to point out that an understanding of the behaviour of a very massive body during gravitational contraction through starlike phases requires the solution of the full dynamic equations of general relativity. Thanks Fowler for suggestions and Feynman and Iben for valuable discussions.
- In 1962 a Center for Relativity headed by A. Schild was established at the University of Texas. Among the scholar who worked at the center in the period 1962-64 there are Kerr, Penrose, Schücking, Robinson, Sachs.

The first Conference of Relativistic Astrophysics

June 1963 : Organization of the First Texas Symposium
Quasi stellar sources and gravitational collapse

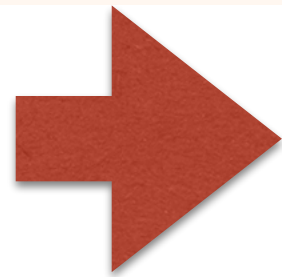


Ivor Robinson, Alfred Schild, Engelbert Schücking and Peter Bergmann sent out an invitation to more than 300 scientists

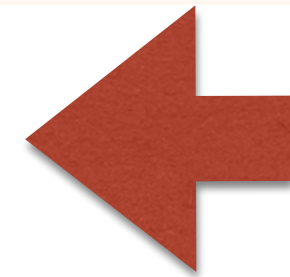
G. Israel recollections: “I recall an ebullient Roy Kerr treating me in my hotel room to a preview of his talk: ‘Pass through this magic ring and -presto!- you're in a completely different universe where radius and mass are negative!’ This was heady stuff, but it seemed to bear no relation to physics...”

But when I presented this at the conference, the astrophysicists did not even bother to listen, much to the annoyance of the relativists present.





ON RELATIVISTIC ASTROPHYSICS



F. HOYLE

St. John's College, Cambridge, England, and
California Institute of Technology, Pasadena, California

WILLIAM A. FOWLER

California Institute of Technology, Pasadena, California

G. R. BURBIDGE AND E. MARGARET BURBIDGE

University of California, San Diego, La Jolla, California

Received October 5, 1963

ABSTRACT

In this paper we have attempted to discuss the relation of massive highly condensed objects to astrophysics in general, rather than only to the radio-source problem. Because situations in which general relativistic effects play a dominant role have not received much attention in astrophysics, a brief review of the relativistic properties of collapsed objects is given in Section I of the paper.

In Section II we have given especial attention to two problems: (1) Galaxies may contain considerable quantities of inert or "hidden" mass, not simply in the form of white dwarfs or neutron stars. (2) Highly collapsed objects may act as energy sources, not just explosively, but over extended time intervals.

The first of these problems has implications both for nucleosynthesis and for the mass-to-light properties of stellar systems. The second appears applicable to a wide range of phenomena, ranging from supernovae in our Galaxy to whole radio galaxies. A continuous source of optical synchrotron electrons appears necessary both in the Crab Nebula and in the jet of M87. We suggest their origin lies in processes that are physically similar and in which considerations of general relativity play a dominant role.

44110. The Universe and its Origin. Herausgegeben von H. Messel, S. T. Butler. New York, St. Martin's Press, 1964. 147 S. Preis \$ 3.75. — B. in Phys. Today **17** Nr. 10 S. 60, Science **145** 1288, Sky Tel. **29** 378.

Gravitativer Kollaps

44111. R. d'E. Atkinson, An energy-momentum tensor for collapsing stars. Proc. National Acad. Sci. USA **51** 723—730. — Der relativistische Energie-Impuls-Tensor für eine ideale Flüssigkeit wird durch Einführung eines den Strahlungsfluß darstellenden Terms in der Weise modifiziert, daß er auf einen Stern großer Masse ($M = 10^8 M_\odot$) im Stadium des schnellen Zusammenbruchs anwendbar wird. Dabei wird auch einem nichtisotropen Druck Rechnung getragen. Die Komponenten des neuen Tensors werden für den Fall sphärischer Symmetrie explizit angegeben.

Henn

44112. A. Balklavs, Superzvaigznes. ZD 1964. gada rudens S. 1—9.

44113. P. G. Bergmann, Gravitational collapse. Phys. Rev. Letters **12** 139—140.

44114. H. Bondi, Gravitational collapse. Nature **202** 275.

44115. H. Bondi, The contraction of gravitating spheres. Proc. Roy. Soc. (A) **281** 39—48. — Die Feldgleichungen der Allgemeinen Relativitätstheorie werden in Schwarzschild-Koordinaten angegeben. Die langsame adiabatische Kontraktion einer Kugel konstanter Dichte wird untersucht, die Teilchenbahnen werden beschrieben. Für die Abhängigkeit des Drucks von der Dichte genügt nicht mehr ein Gesetz mit dem Exponenten 4/3 der einfachen Newtonschen Theorie, sondern eine Abhängigkeit mit einem etwas höheren Exponenten. Zusätzlich wird der Fall untersucht, in dem das kontrahierende System Strahlung aussendet.

Oster

44116. W. B. Bonnor, Gravitational collapse. Nature **204** 868.

44117. G. Boročki, Keine Supergalaxien, sondern Supersterne. Vasiona **12** 32 (serb.-kroat.).

44118. S. Bowyer, E. T. Byram, T. A. Chubb, H. Friedman, Possible neutron star. Harv Card 1643.

* * S. Bowyer, E. T. Byram, T. A. Chubb, H. Friedman, Rocket astronomy studies of the Crab nebula: Röntgen-ray sources in space and the neutron-star concept. Vgl. Ref. 13108.

44119. H. A. Buchdahl, A relativistic fluid sphere resembling the Emden polytrope of index 5. ApJ **140** 1512—1516.

44120. S. Chandrasekhar, The dynamical instability of gaseous masses approaching the Schwarzschild limit in general relativity. ApJ **140** 417—433. — Oszillationen als Folge kleiner, linear behandelter Störungen der Ausgangsgleichungen radialsymmetrischer Massen führen zu charakteristischen Abweichungen von dem analogen Fall der Newtonschen Mechanik, und eine Instabilität setzt ein, ehe der Grenzwert 4/3 für das Verhältnis der spezifischen Wärmen in der Zustandsgleichung erreicht wird, und lange vor dem Erreichen des Schwarzschildschen Grenzradius. Es werden Tabellen für die numerische Behandlung verschiedener Fälle gegeben.

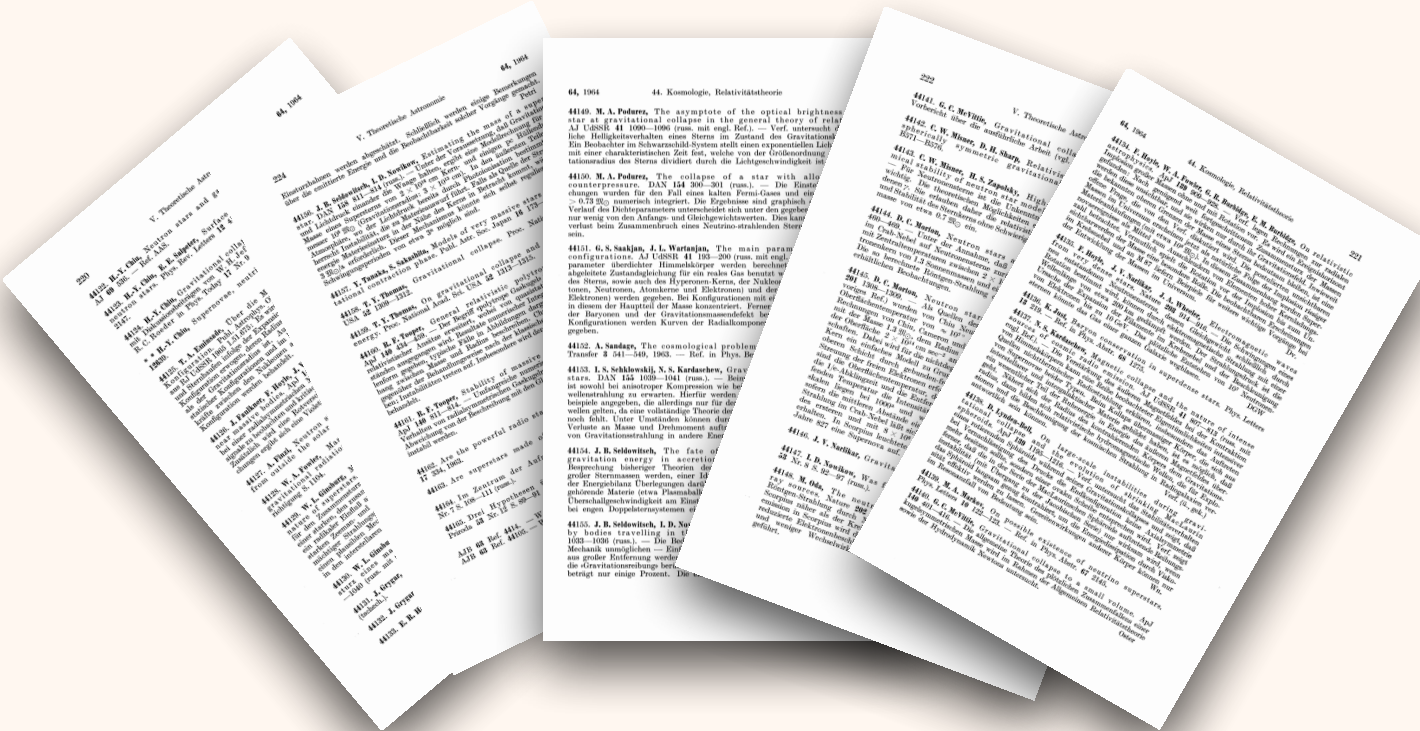
Dv.

44121. S. Chandrasekhar, Dynamical instability of gaseous masses approaching the Schwarzschild limit in general relativity. Phys. Rev. Letters **12** 114—116. — Ref. in Phys. Abstr. **67** 1428.

Astronomischer Jahresbericht 64 (1964)

V Theoretical Astronomy

§ Cosmology, General Relativity, Gravitational Collapse



Astronomischer Jahresbericht 65 (1965)

V Theoretical Astronomy

§ Cosmology, Relativistic Astrophysics, Relativistic properties of stars

Astronomischer Jahresbericht 67 (1967)

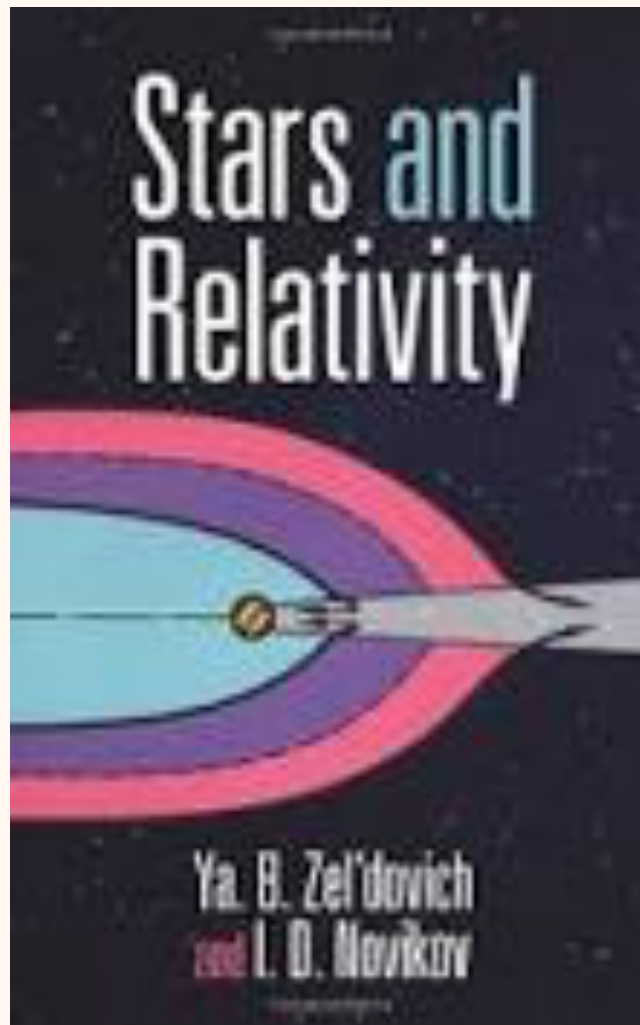
VI Theoretical Astrophysics

§ Gravitational Collapse, Neutron stars

Astronomischer Jahresbericht 69 (1969)

VI Theoretical Astrophysics

§ General Theoretical Problems of Astrophysics, Gravitational instability, Neutrino Astronomy, X Ray- and Gamma Ray Astronomy, Frequency and origin of elements

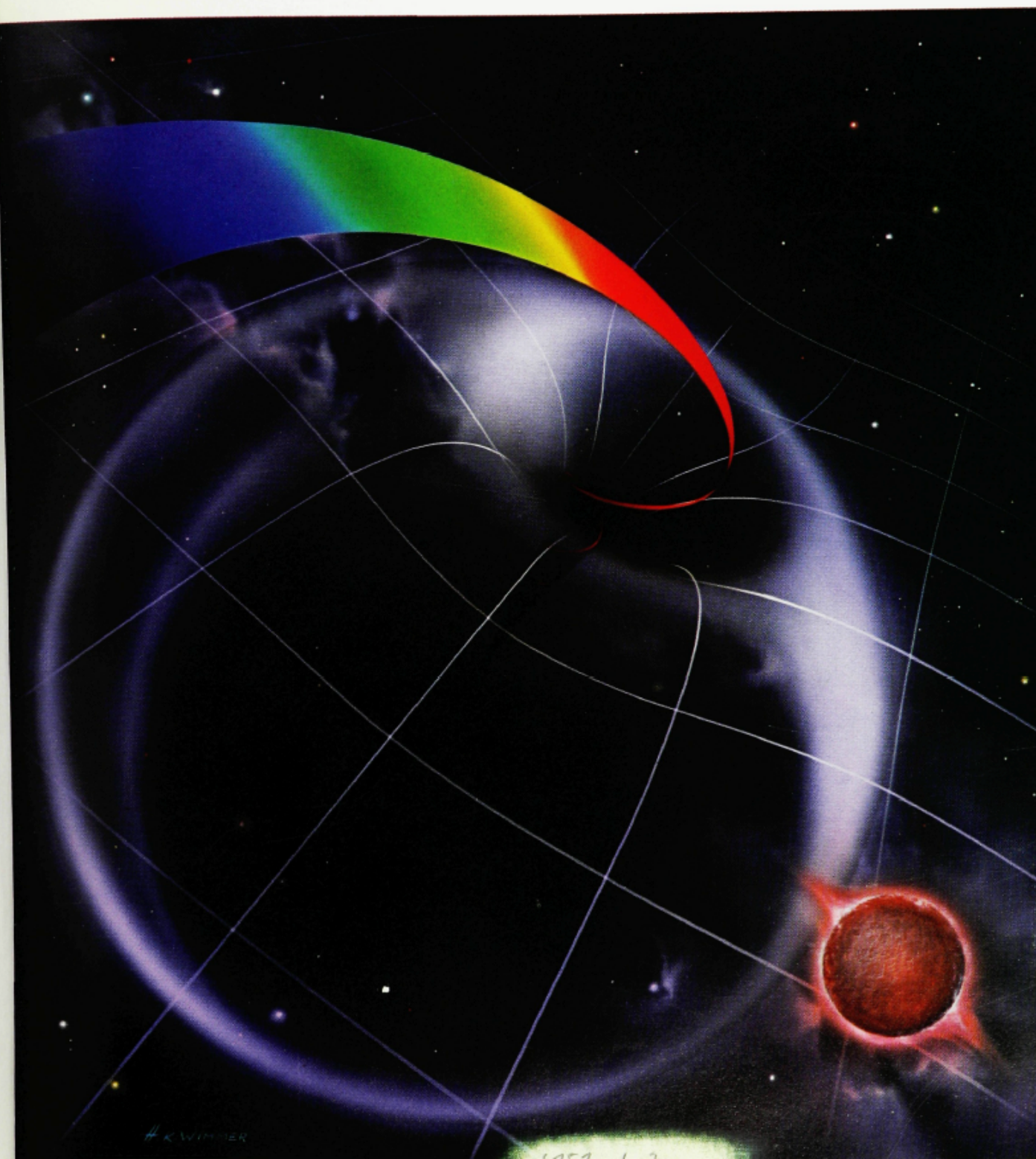


“Tomorrow is Yesterday”
***Star Trek* episode, first**
aired: 26 January 1967

physics today

JANUARY 1971

Introducing the black hole



Introducing the black hole

According to present cosmology, certain stars end their careers in a total gravitational collapse that transcends the ordinary laws of physics.

Remo Ruffini and John A. Wheeler

The quasistellar object, the pulsar, the neutron star have all come onto the scene of physics within the space of a few years. Is the next entrant destined to be the black hole? If so, it is difficult to think of any development that could be of greater significance. A black hole, whether of "ordinary size" (approximately one solar mass, $1 M_{\odot}$), or much larger (around $10^6 M_{\odot}$ to $10^{10} M_{\odot}$, as proposed in the nuclei of some galaxies) provides our "laboratory model" for the gravitational collapse, predicted by Einstein's theory, of the universe itself.

A black hole is what is left behind after an object has undergone complete gravitational collapse. Spacetime is so strongly curved that no light can come

nitely high density, and he himself will be torn apart eventually by indefinitely increasing tidal forces. No restraining force whatsoever has the power to hold him away from this catastrophe, once he crossed a certain critical surface known as the "horizon." The final collapse occurs a finite time after the passage of this surface, but it is inevitable. Time and space are interchanged inside a black hole in an unusual way; the direction of increasing proper time for the observer is the direction of decreasing values of the coordinate r . The observer has no more power to return to a larger r value than he has power to turn back the hands on the clock of life itself. He can not even stay where he is, and for a simple reason: no one has

maximum rate, or near the maximum rate, allowed for a black hole ("surface velocity" equal to speed of light). Roger Penrose³ has shown that a particle coming from a distance into the immediate neighborhood of a black hole (the "ergosphere") can extract energy from the black hole. Demetrios Christodoulou⁴ has shown that the total mass-energy of a black hole can be split into three parts,

$$E^2 = m_{ir}^2 + L^2/4m_{ir}^2 + p^2$$

The first part is "irreducible" (left constant in "reversible transformations"; always increased in "irreversible transformations") and the second and third parts (arising from a rotational angular momentum L and a linear momentum

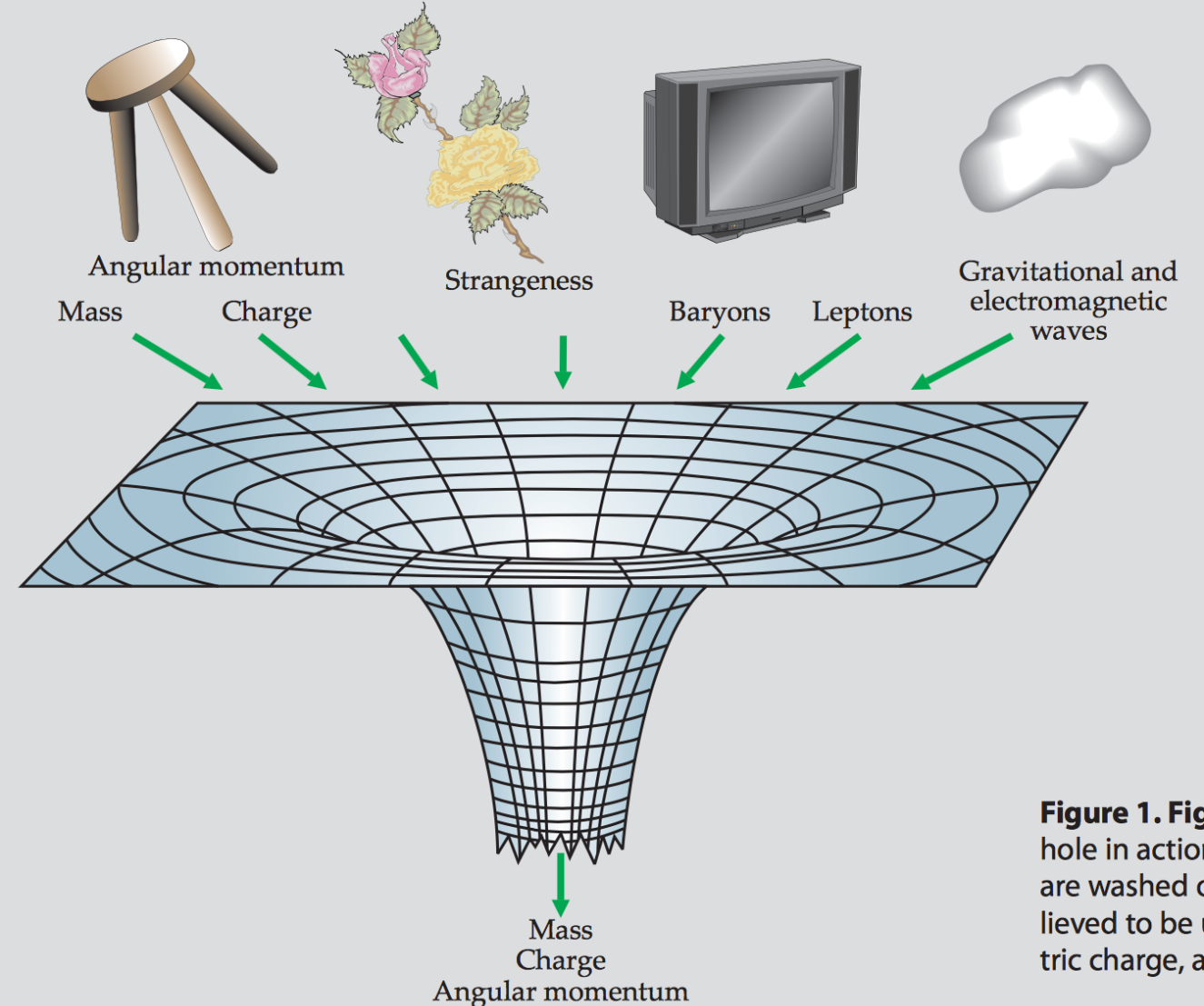


Figure 1. Fig
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