A Curious Story of Quantum Gravity in the Ultraviolet

A Century of General Relativity

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Quantum Gravity

Often repeated statement:

"Einstein's theory of General Relativity is incompatible with quantum mechanics."

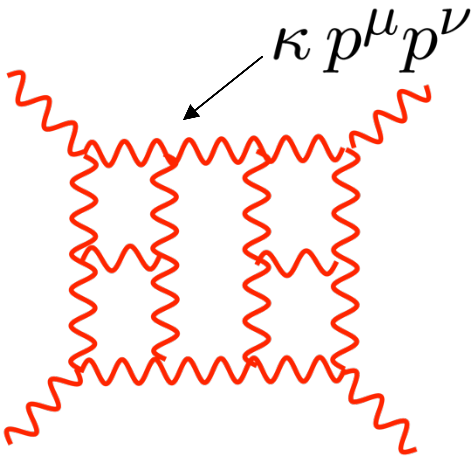
To a large extent this is based on another often repeated statement:

"All point-like quantum theories of gravity are ultraviolet divergent and non-renormalizable."

Where do these statements come from and are they true?

Ultraviolet Behavior

$$\kappa = \sqrt{32\pi G_N}$$ ⟵ **Dimensionful coupling**



Einstein gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(\kappa p_j^\mu p_j^\nu)}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g p_j^\nu)}{\text{propagators}}$$

Extra powers of loop momenta in numerator:
Integrals are badly behaved in the UV.

Origin of simplistic statement that all point-like theories of gravity must be ultraviolet divergent.

Are we sure there must be divergence?
Cancellations between pieces?

Test case: $N = 8$ Supergravity

The best theories to look at are supersymmetric theories.

Supersymmetry relates bosons (forces) and fermions (matter)

We first consider $N = 8$ supergravity.

Einstein gravity + 254 other physical states

Reasons to focus on $N \geq 4$ supergravity:
- With more supersymmetry expect better UV properties.
- High symmetry implies technical simplicity.

In the late 70's and early 80's supergravity was seen as the primary means for unifying gravity with other forces.

Ferrara, Freedman, van Nieuwenhuizen

UV Finiteness of $N = 8$ Supergravity?

If $N = 8$ supergravity is perturbatively finite it would imply a new symmetry or non-trivial dynamical mechanism.

Such a mechanism would have a fundamental impact on our understanding of gravity.

Of course, perturbative finiteness is not the only issue for consistent gravity:

- **Nonperturbative completions?**
- **High-energy behavior?**
- **Realistic models?**

Here we are trying to answer a simple question:

Is $N = 8$ supergravity ultraviolet finite to all order of perturbation theory? Yes, or no?

Opinions from the 80's

If certain patterns that emerge should persist in the higher orders of perturbation theory, then … $N = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops. Green, Schwarz, Brink, (1982)

It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions… The final word on these issues may have to await further explicit calculations.

Marcus, Sagnotti (1985)

The idea that *all* supergravity theories diverge at 3 loops has been widely accepted wisdom for over 25 years, with a only handful of contrarian voices.

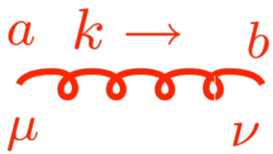
Status of UV Divergences

… it is not clear that general relativity, when combined with various other fields in supergravity theory, can not give a sensible quantum theory. Reports of the death of supergravity are an exaggeration. One year everyone believed that supergravity was finite. The next year the fashion changed and everyone said that supergravity was bound to have divergences even though none had actually been found.

— Stephen Hawking, 1994

Feynman Diagrams

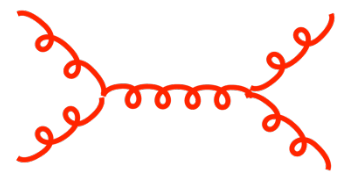
Every graduate student in particle physics learns how to calculate with Feynman diagrams.

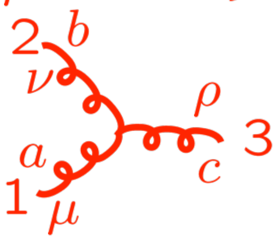


$$\frac{i}{k^2 + i\epsilon}$$

Feynman propagator

gluon scattering

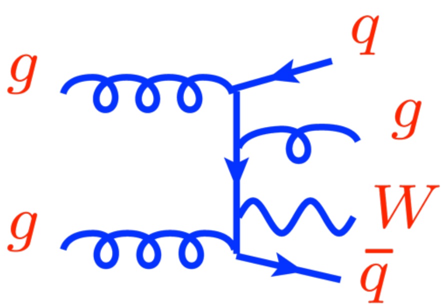




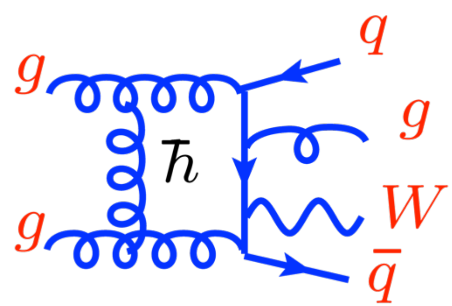
Coupling constant

$$-g f^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + \eta_{\nu\rho}(k_2-k_3)_\mu + \eta_{\rho\mu}(k_3-k_1)_\nu)$$

color factor



Tree – Leading Order (LO)



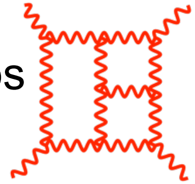
Loop – Next-To-Leading Order (NLO)
Quantum correction

Feynman diagrams give us a perturbative expansion in Planck's constant $\hbar$

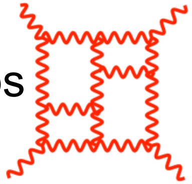
Gravity similar but much much more complicated.

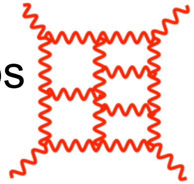
Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

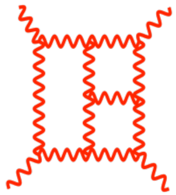
More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

With Modern Ideas

Z B, Carrasco, Dixon, Johansson, Roiban

For $N = 8$ supergravity.

3 loops  ~10 TERMS

4 loops  $\sim 10^2$ TERMS

Much more manageable!

5 loops  $\sim 10^4$ TERMS (Not yet done—1000 diagrams)

We now have the ability to settle the 35 year debate and determine the true UV behavior gravity theories.

Where is First Potential $D = 4$ UV Divergence?

3 loops **$N = 8$**	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops **$N = 8$**	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops **$N = 8$**	Howe and Stelle (2003)	✗
7 loops **$N = 8$**	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
3 loops **$N = 4$**	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops **$N = 5$**	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops **$N = 4$**	Vanhove and Tourkine (2012)	✓
9 loops **$N = 8$**	Bekovits, Green, Russo and Vanhove (2009)	✗

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

Don't bet on this now!

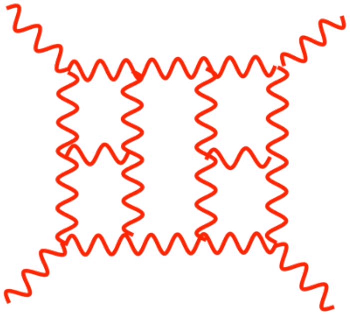
"Enhanced cancellations"

Weird structure. Quantum anomaly behind divergence.

retracted

- **Conventional wisdom: divergence are expected at some high loop order.**
- **So far, *every* specific prediction of divergences in pure supergravity has either been wrong or missed crucial details.**

New Structures?



Might there be a new unaccounted structure in gravity theories that suggests the UV might be is tamer than conventional arguments suggest?

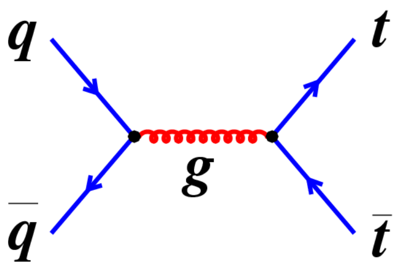
Yes!

Scattering Amplitudes

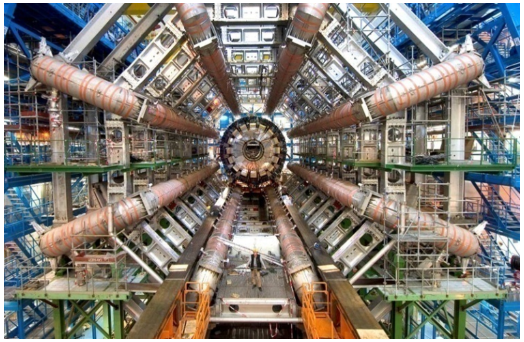
To understand how we attack the question of gravity in the UV first need a detour through scattering amplitudes.

Scattering amplitudes

Scattering of elementary particles is fundamental to our ability to unravel microscopic laws of nature.



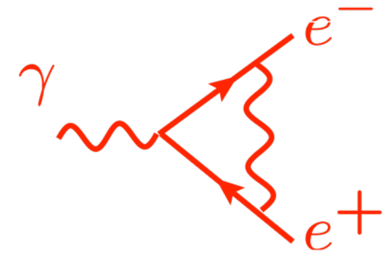
Smooth running of the Large Hadron Collider at CERN emphasizes importance of scattering amplitudes.



Vast subject.
Here we give some examples of advances of past few years relevant for gravity.

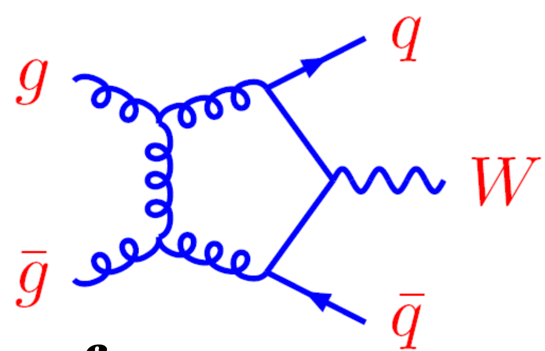
State-of-the-Art Feynman Diagram Calculations

In 1948 Schwinger computed anomalous magnetic moment of the electron.



In 2010 typical 1-loop modern example:

$$gg \to W + 2 \text{ jets}$$



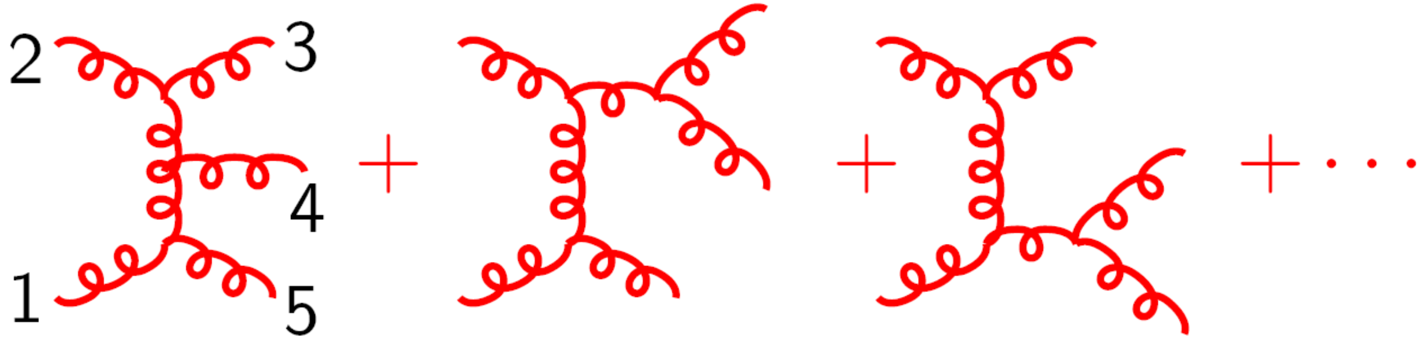
g **: gluon -- carrier of strong force**
q**: quark -- matter**
W, Z **: Weak boson carrier of weak interaction**

60 years later at 1 loop only 2 or 3 legs more than Schwinger if we use traditional Feynman diagrams.

Tree-level example: Five gluons

Force carriers in QCD are gluons. Similar to photons of QED except they self interact.

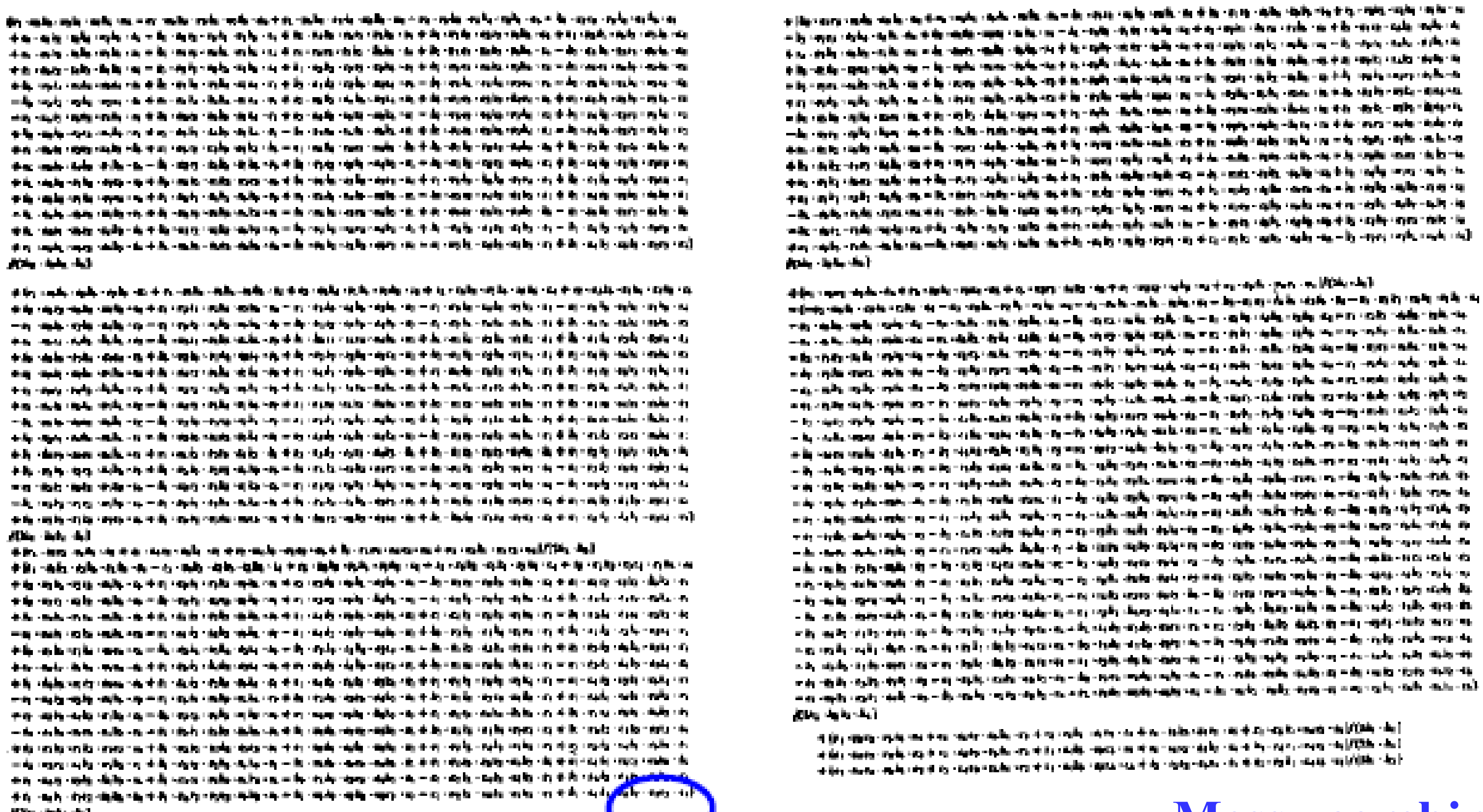
Consider the five-gluon amplitude:



Used in calculation of *pp* ⟶ 3 jets at CERN

If you evaluate this following textbook Feynman rules you find…

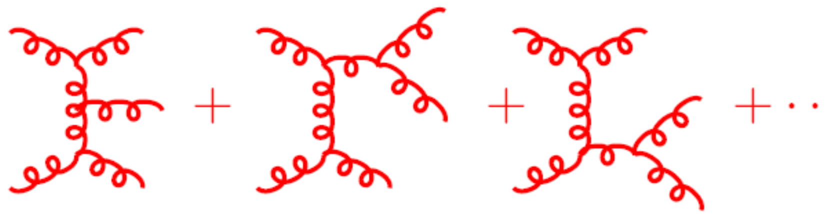
Result of evaluation (actually only a small part of it):



$k_1 \cdot k_4 \, \varepsilon_2 \cdot k_1 \, \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5$

Messy combination of momenta and gluon polarization vectors.

Reconsider Five-Gluon Tree



With a little Chinese magic, i.e. helicity states:

Xu, Zhang and Chang and many others

$$A_5^{\text{tree}}(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$$A_5^{\text{tree}}(1^-, 2^+, 3^-, 4^+, 5^+) = i\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

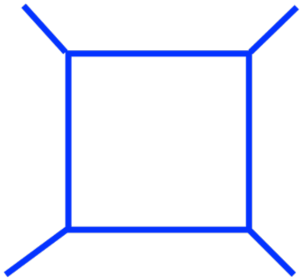
Use a better organization of color charges:

$$\mathcal{A}_5 = \sum_{\text{perms}} \text{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5})\, A_5(1,2,3,4,5)$$

Motivated by the color organization of open string amplitudes.

Mangano and Parke

Example of loop difficulty



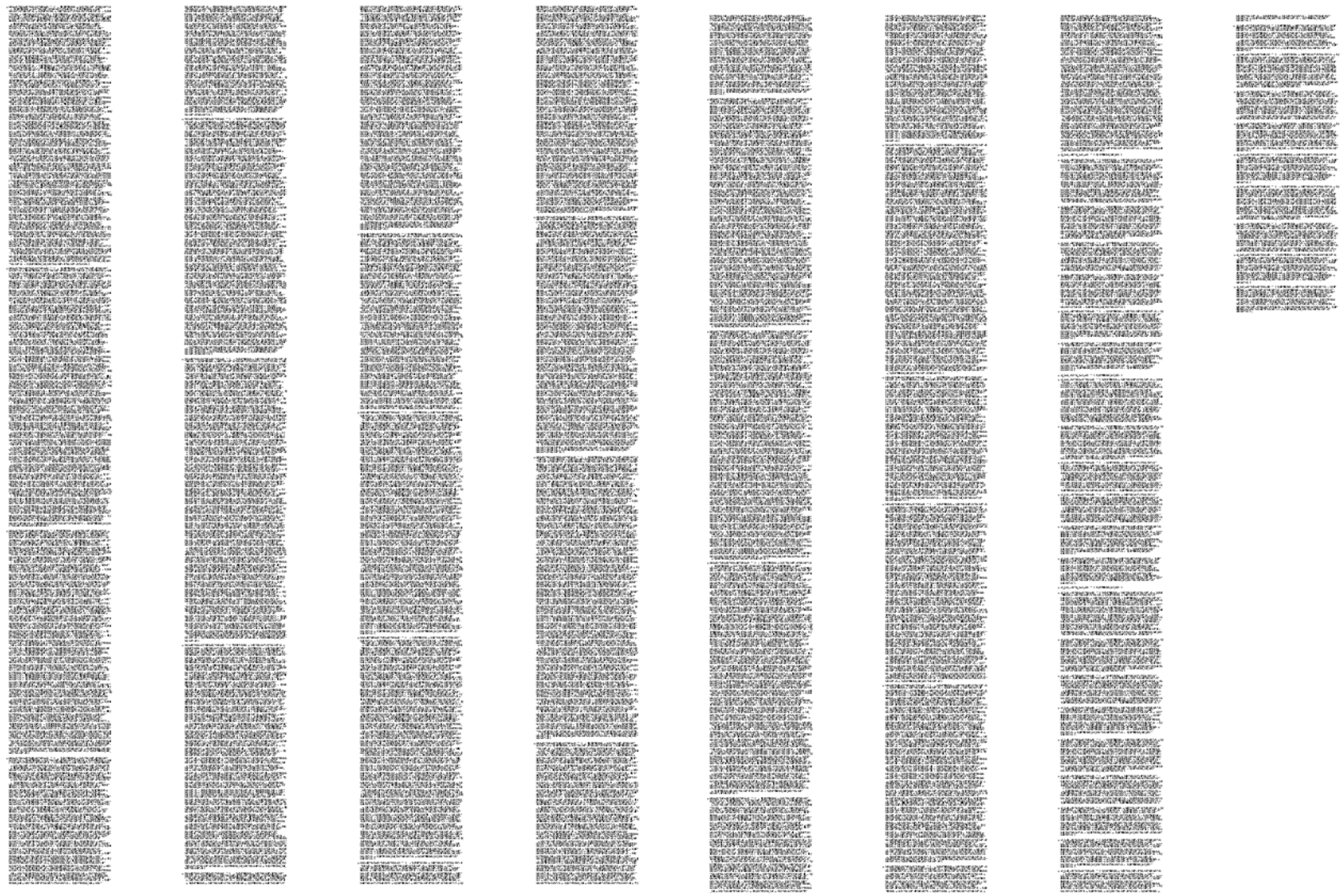
Consider a tensor integral:

$$\int \frac{d^{4-2\epsilon}\ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu\, \ell^\nu\, \ell^\rho\, \ell^\lambda}{\ell^2\,(\ell-k_1)^2\,(\ell-k_1-k_2)^2\,(\ell+k_4)^2}$$

Note: this is trivial on modern computer. Non-trivial for larger numbers of external particles.

Evaluate this integral via Passarino-Veltman reduction. Result is …

Result of performing the integration



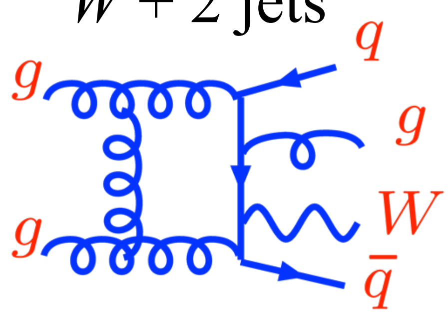
Calculations explode for larger numbers of particles or loops. Clearly, there should be a better way!

$W + n$ Jet NLO Calculations for the LHC

Suppose you said "I'm going not shave until I finish calculating"

$W + 1$ jet

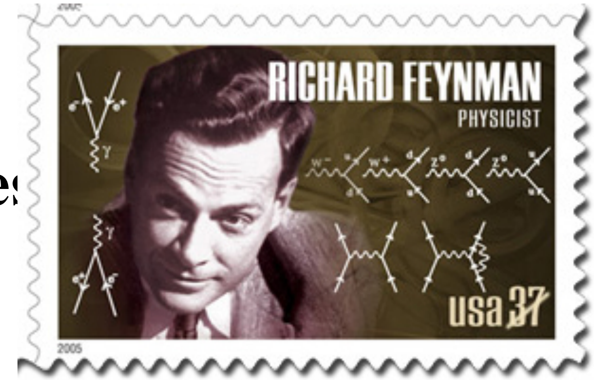
$W + 2$ jets



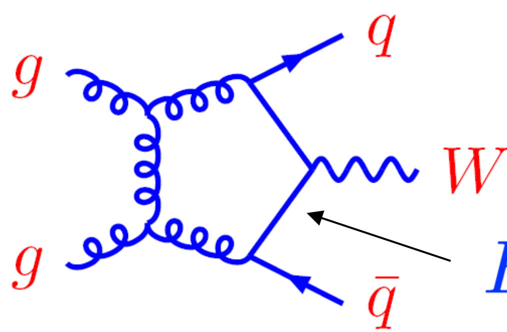
$W + 3$ jets
Still no Feynman diagram calculation

Why are Feynman diagrams difficult for high-loop or high-multiplicity processes?

- **Vertices and propagators involve unphysical gauge-dependent off-shell states. An important origin of the complexity.**



$$\int \frac{d^3\vec{p}\, dE}{(2\pi)^4}$$



$$E^2 - \vec{p}^{\,2} \neq m^2$$

Individual Feynman diagrams unphysical

Einstein's relation between momentum and energy violated in the loops. Unphysical states! Not gauge invariant.

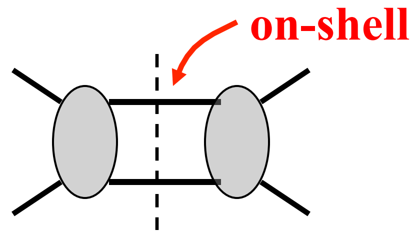
- **Use gauge invariant on-shell physical states.**
- **On-shell formalism.**
- **Don't violate Einstein's relation!** ZB, Dixon, Dunbar, Kosower (1998)

Modern Unitarity Method

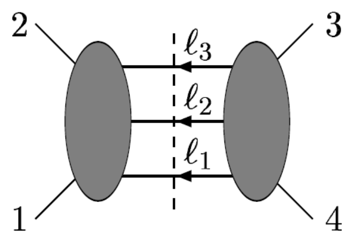
$$E^2 = p^2 + m^2$$

ZB, Dixon, Dunbar and Kosower

Two-particle cut:

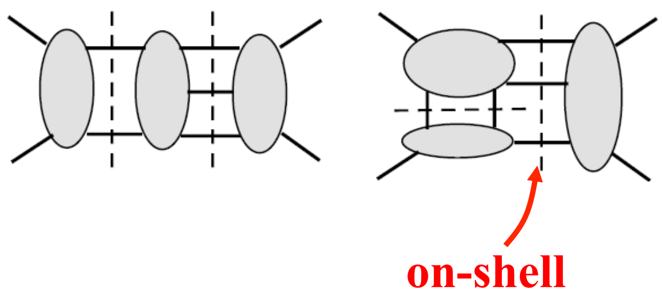


Three-particle cut:



- **Systematic assembly of complete amplitudes from other amplitudes.**
- **Works for any number of particles or loops.**

Generalized unitarity as a practical tool.

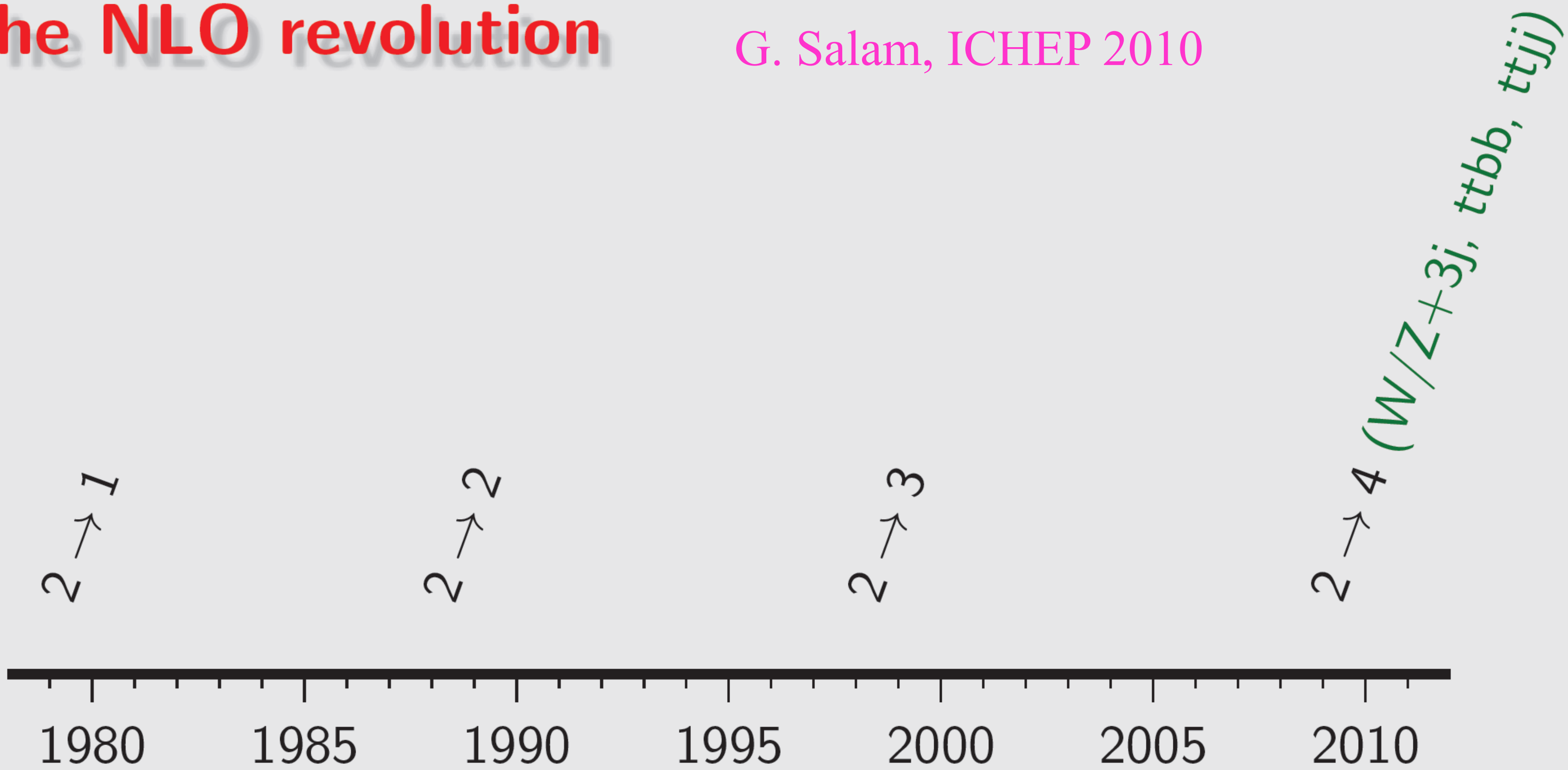


ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger
and many others

Reproduces Feynman diagrams except intermediate steps of calculation based on physical quantities *not* unphysical ones.

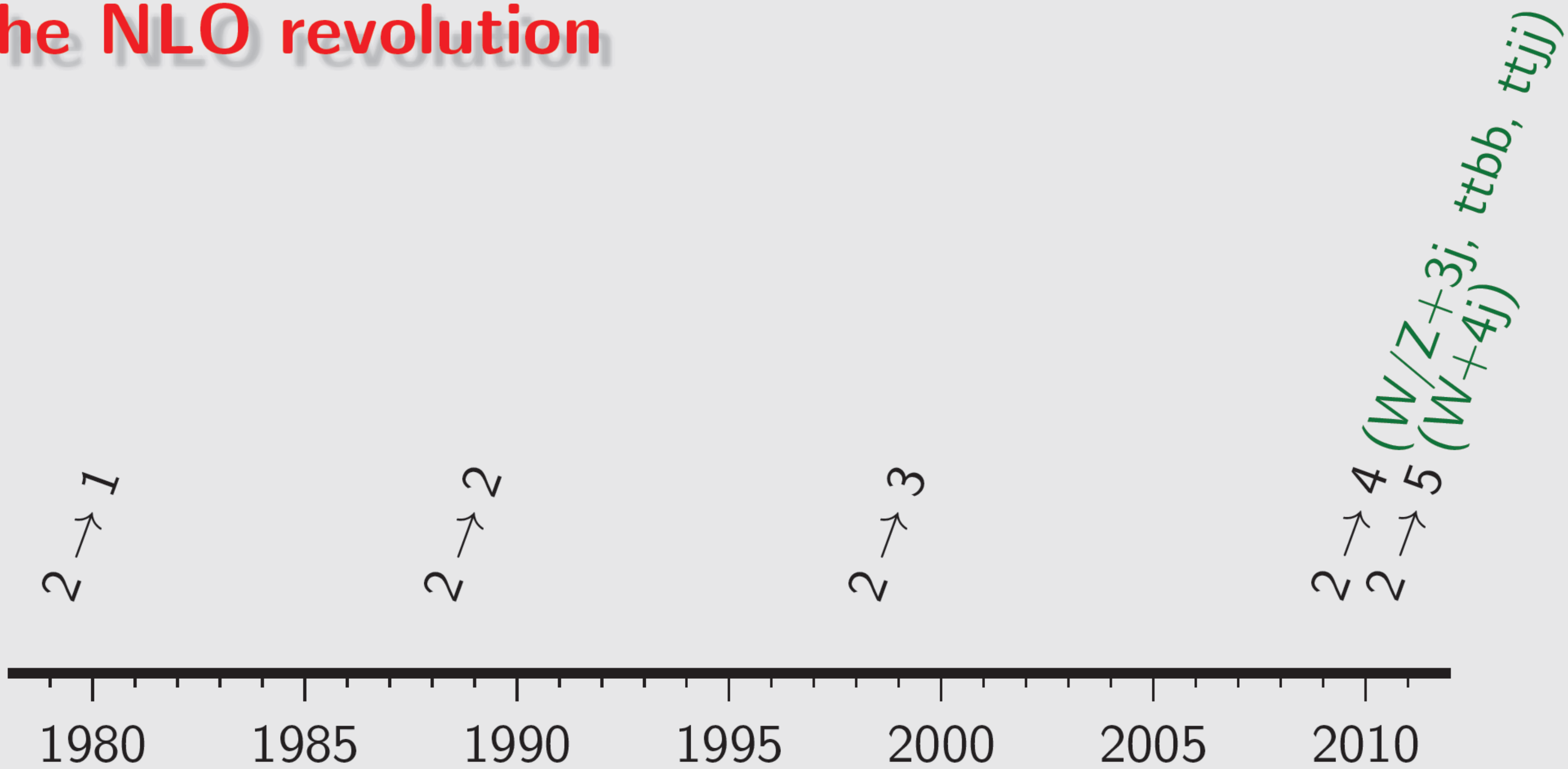
The NLO revolution

G. Salam, ICHEP 2010



2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]
2009: NLO $W+3j$ [BlackHat: Berger et al] [unitarity]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2010: NLO $Z+3j$ [BlackHat: Berger et al] [unitarity]

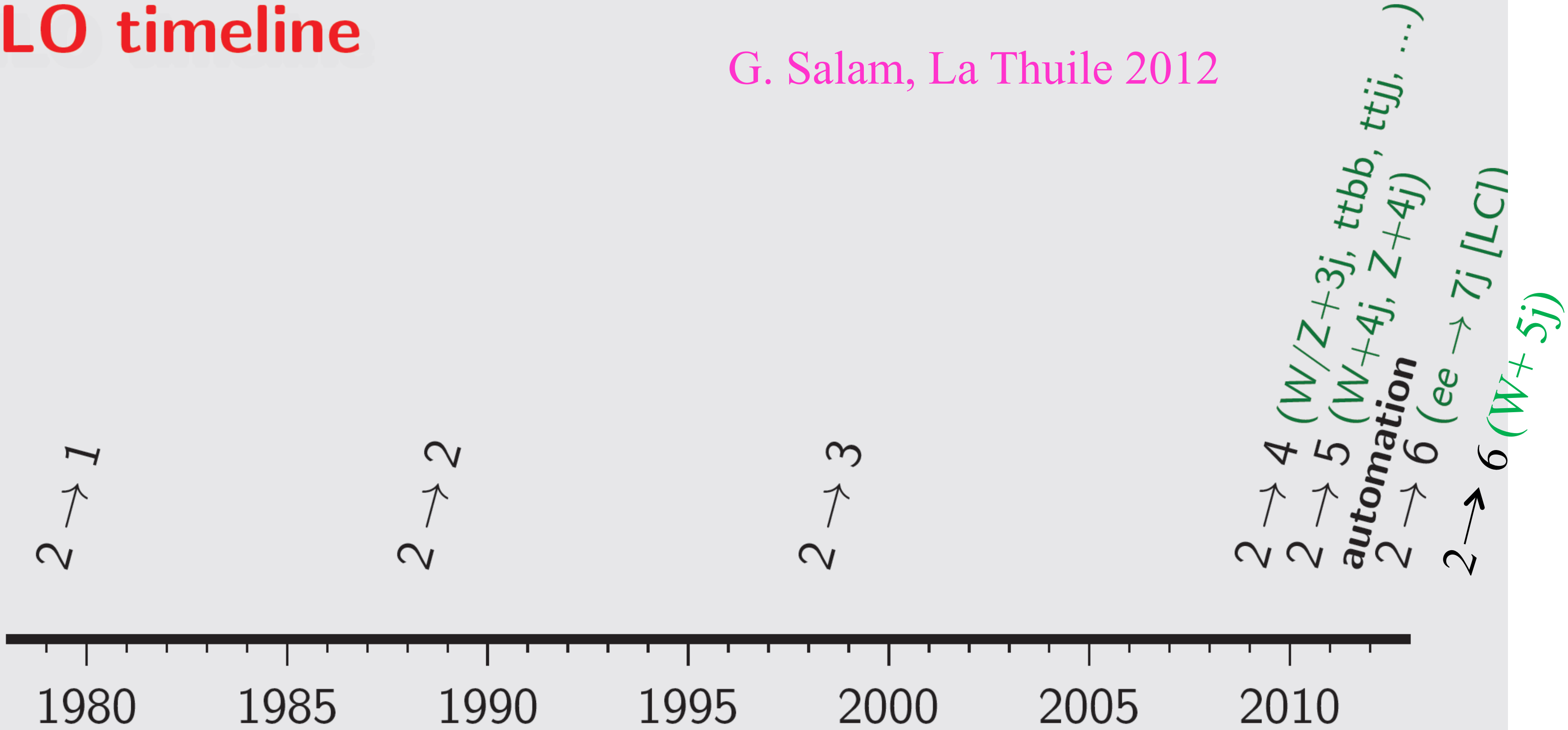
The NLO revolution



2010: NLO $W{+}4j$ [BlackHat: Berger et al, preliminary] [unitarity]

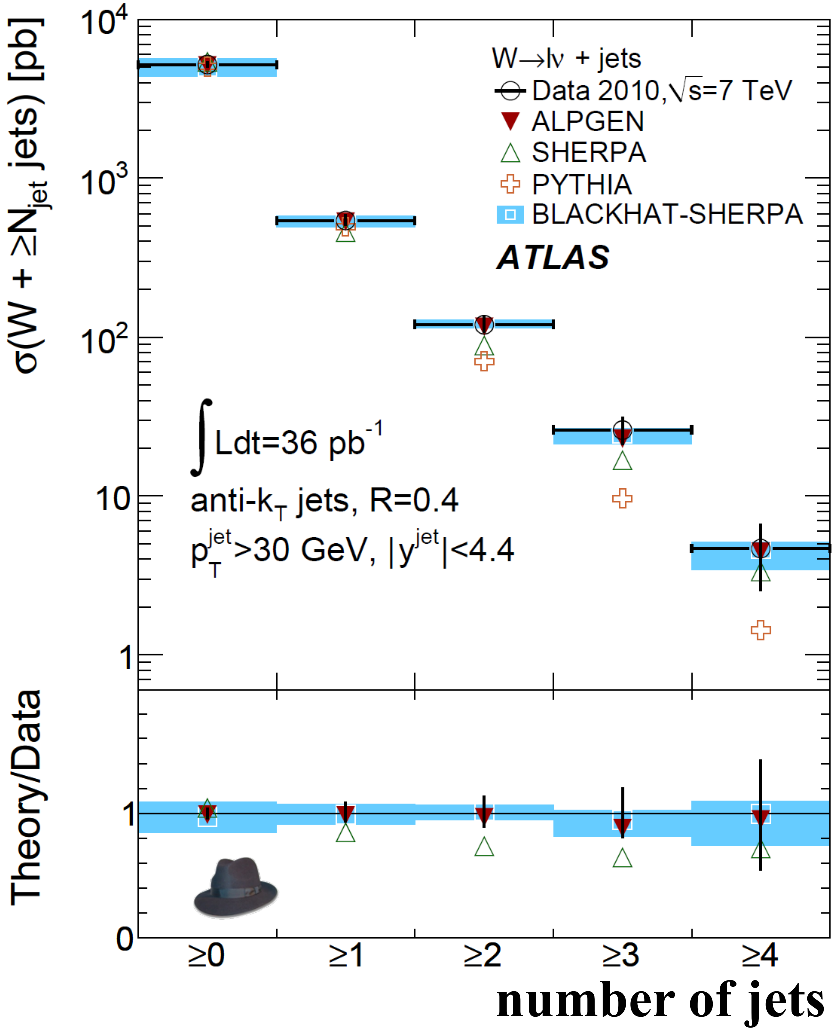
NLO timeline

G. Salam, La Thuile 2012



2010: NLO W+4j [BlackHat+Sherpa: Berger et al] [unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al] [unitarity]
2011: NLO Z+4j [BlackHat+Sherpa: Ita et al] [unitarity]
2011: NLO 4j [BlackHat+Sherpa: Bern et al] [unitarity]
2011: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011: first automation [GoSam: Cullen et al] [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]
2013: NLO W+5j [BlackHat+Sherpa: Bern et al] [unitarity]

ATLAS Comparison against NLO QCD



Powerful experimental confirmation of NLO approach.

It works out of the box. No tuning! Based on fundamental theory.

Theory predictions from BlackHat NLO QCD used to aid search for supersymmetry.

***W* +1, 2, 3, 4 jets inclusive**

Back to Gravity

Gravity vs Gauge Theory

Consider the Einstein gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R$$

curvature

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

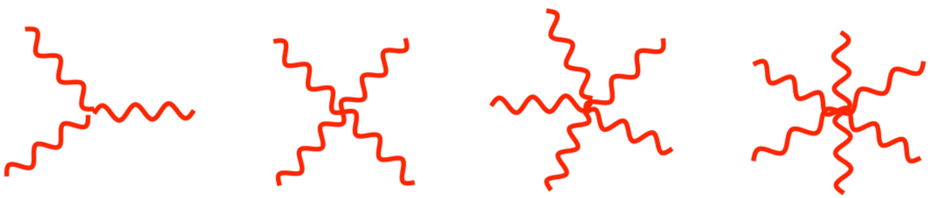
$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

metric

Flat-space metric

graviton field

Infinite number of complicated interactions

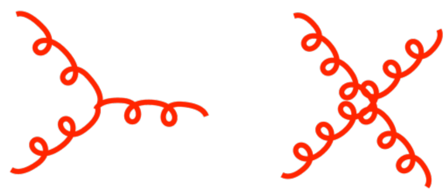


$+ \cdots$

terrible mess

Compare to gauge-theory Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2} F^2$$



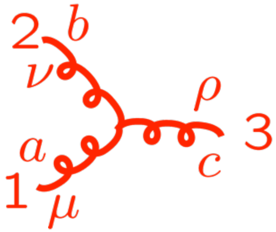
Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Does not look related!

Three Vertices

Standard Feynman diagram approach.



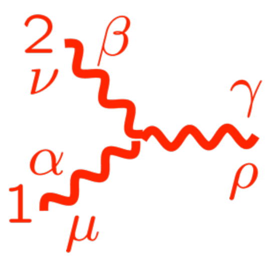
Three-gluon vertex:

$$V_{3\,\mu\nu\sigma}^{abc} = -gf^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho+\eta_{\nu\rho}(k_1-k_2)_\mu+\eta_{\rho\mu}(k_1-k_2)_\nu)$$

Three-graviton vertex:

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

$$\begin{aligned}
&G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \\
&\quad \mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\
&\qquad + P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) \\
&\qquad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) \\
&\qquad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$



About 100 terms in three vertex

Naïve conclusion: Gravity is a nasty mess.

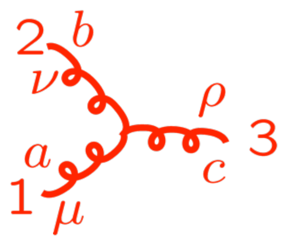
Simplicity of Gravity Amplitudes

People were looking at gravity the wrong way. On-shell viewpoint much more powerful.

On-shell **three vertices contains all information:**

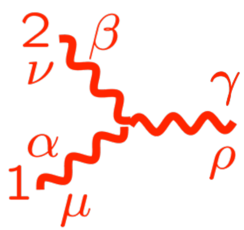
$$k_i^2 = 0$$

Yang-Mills gauge theory:



$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

Einstein gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$
$$\times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

"square" of Yang-Mills vertex.

Using modern methods, any gravity scattering amplitude constructible solely from *on-shell* 3 vertex. Higher-point vertices irrelevant!

Harmony is revealed!

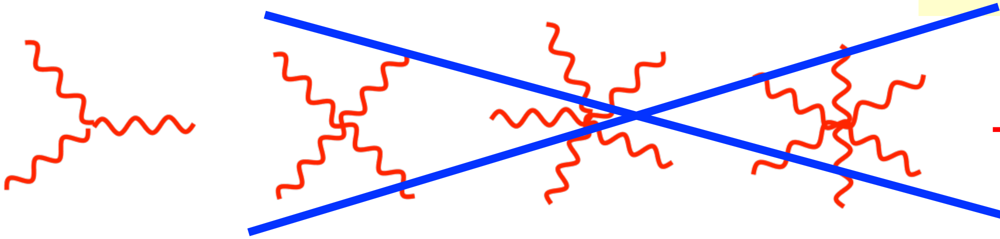
Gravity vs Gauge Theory

Consider the Einstein gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad \underbrace{g_{\mu\nu}}_{\text{metric}} = \underbrace{\eta_{\mu\nu}}_{\text{flat metric}} + \kappa \underbrace{h_{\mu\nu}}_{\text{graviton field}}$$

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

Infinite number of irrelevant interactions!

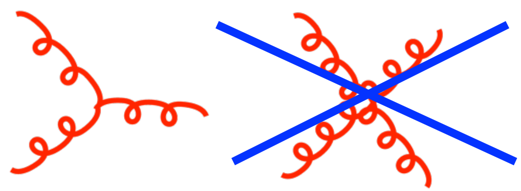


$+ \cdots$

Simple relation to gauge theory

Compare to gauge-theory Lagrangian on which QCD is based.

$$L_{\text{YM}} = \frac{1}{g^2} F^2$$



Only three-point interactions

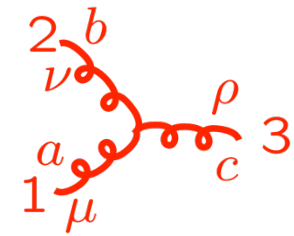
Gravity seems ~~so much~~ no more complicated than gauge theory.

Does ~~not~~ look related!

Duality Between Color and Kinematics

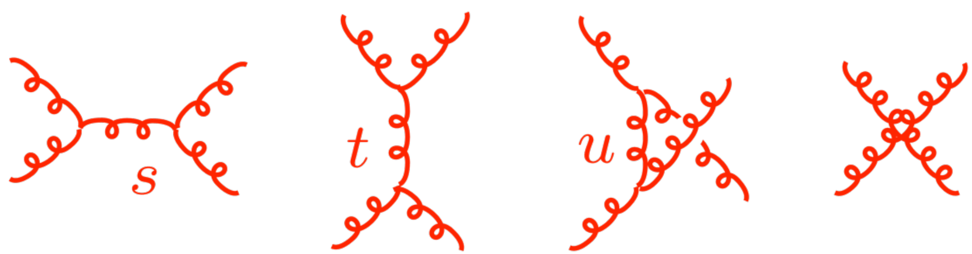
coupling constant **color factor** **momentum dependent kinematic factor**

$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = i f^{abc} T^c$

Jacobi Identity $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.

$$\mathcal{A}_4^{\text{tree}} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

$s = (k_1 + k_2)^2$ $u = (k_1 + k_3)^2$

$t = (k_1 + k_4)^2$

Color factors satisfy Jacobi identity: $c_u = c_s - c_t$

Numerator factors satisfy similar identity: $n_u = n_s - n_t$

Color and kinematics sing the same tune!

Duality Between Color and Kinematics

Consider five-point tree amplitude:

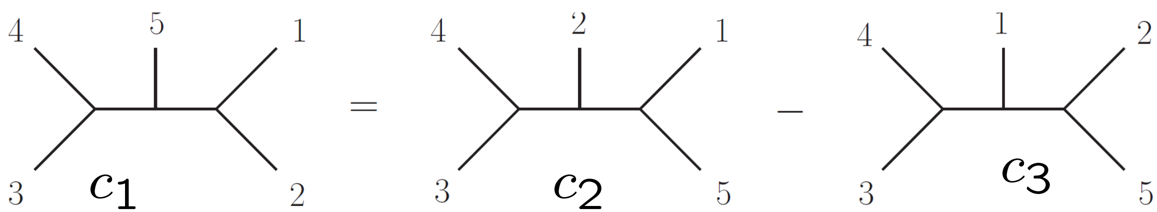
ZB, Carrasco, Johansson (BCJ)

color factor

kinematic numerator factor

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_i^2}$$

Feynman propagators



$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 + c_2 + c_3 = 0 \;\Leftrightarrow\; n_1 + n_2 + n_3 = 0}$$

Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.

Recent progress on unraveling relations.

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer;
Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer
O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White

Higher-Point Gravity and Gauge Theory

ZB, Carrasco, Johansson

color factor

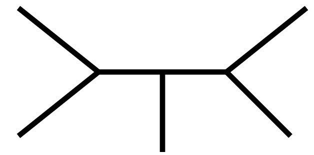
kinematic numerator factor

QCD: $$\mathcal{A}_n^{\text{tree}} = i g^{n-2} \sum_i \frac{c_i \, n_i}{D_i}$$

Feynman propagators

Einstein Gravity: $$\mathcal{M}_n^{\text{tree}} = i \kappa^{n-2} \sum_i \frac{n_i^2}{D_i}$$

$$n_i \sim k_4 \cdot k_5 \, k_2 \cdot \varepsilon_1 \, \varepsilon_2 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$



sum over diagrams with only 3 vertices

Same kinematic numerators appear in gravity and gauge theory!

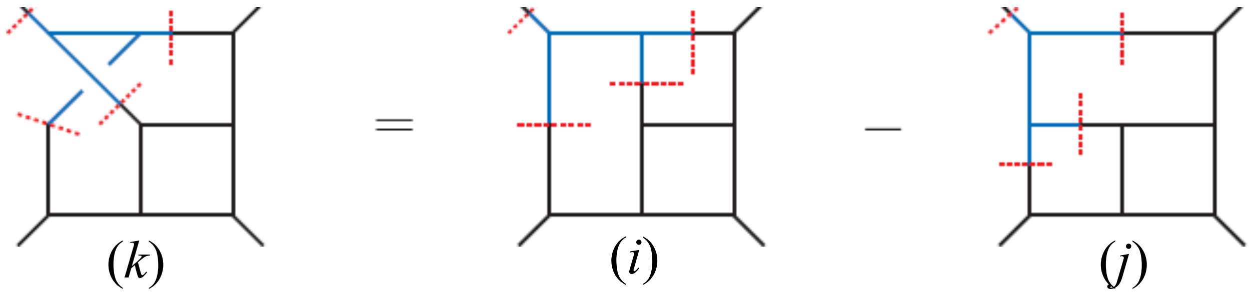
Gravity and QCD kinematic numerators sing same tune!

Cries out for a unified description of the sort given by string theory.

BCJ

Gravity loop integrands are free!

Ideas conjectured to generalize to loops:



color factor

$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory → gravity theory

simply take

color factor → kinematic numerator

$$c_k \rightarrow n_k$$

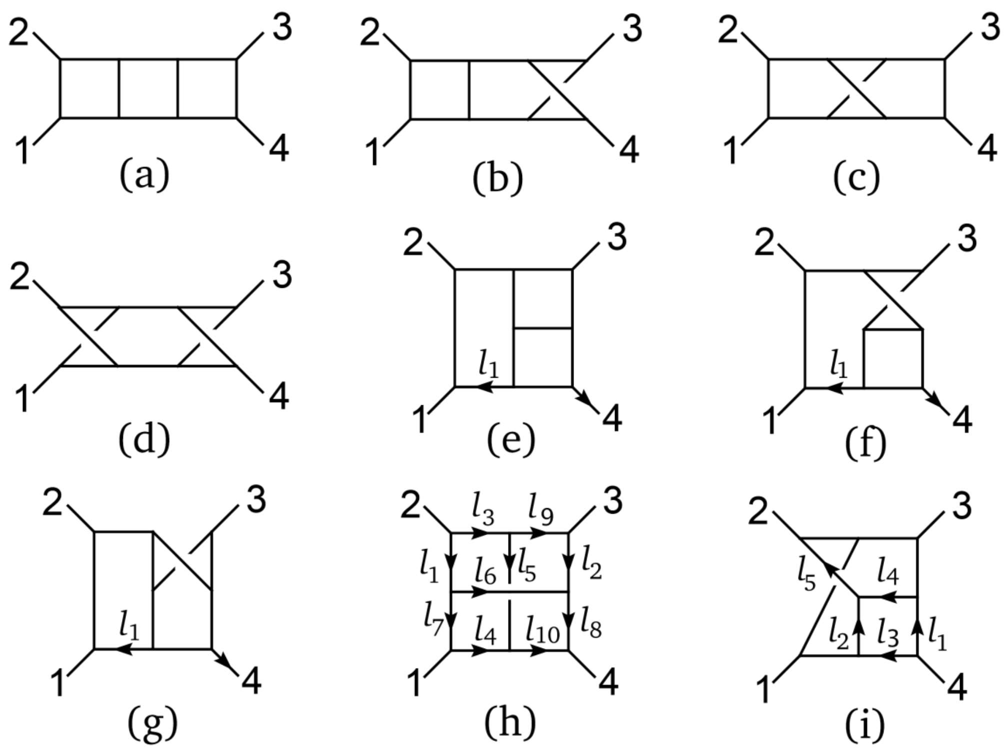
Gravity loop integrands follow from gauge theory!

Applications of On-shell Methods to Quantum Gravity

$N = 8$ Supergravity Three-Loop Result

ZB, Carrasco, Dixon, Johansson, Kosower, Roiban; hep-th/0702112
ZB, Carrasco, Dixon, Johansson, Roiban arXiv:0808.4112 [hep-th]

Obtained via on-shell unitarity method:



Three-loop is not only ultraviolet finite it is "superfinite"—cancellations beyond those needed for finiteness in four dimensions.

UV finite for $D < 6$

More Recent Opinion

In 2009 Bossard, Howe and Stelle had a careful look at the question of how much supersymmetry can tame UV divergences.

In particular … suggest that maximal supergravity is likely to diverge at four loops in $D = 5$ and at five loops in $D = 4$ …

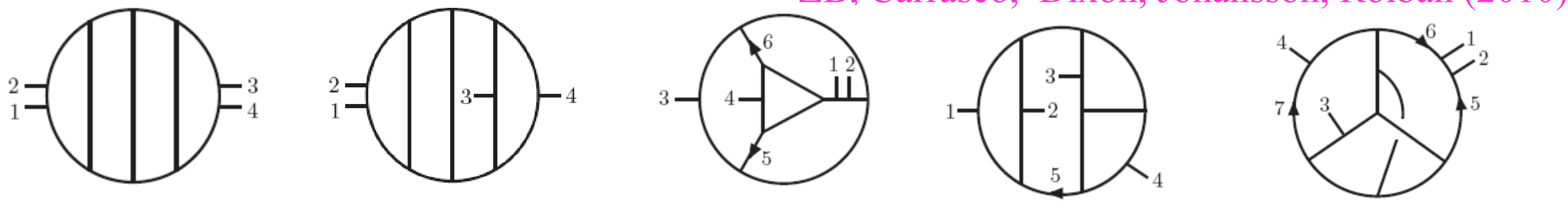
Bossard, Howe, Stelle (2009)

Bottles of wine were at stake!

We had tools to collect the wine.

$N = 8$ Supergravity Four-Loop Calculation

ZB, Carrasco, Dixon, Johansson, Roiban (2010)



50 distinct planar and non-planar diagrammatic topologies

"I'm not shaving until we finish the calculation"
— John Joseph Carrasco

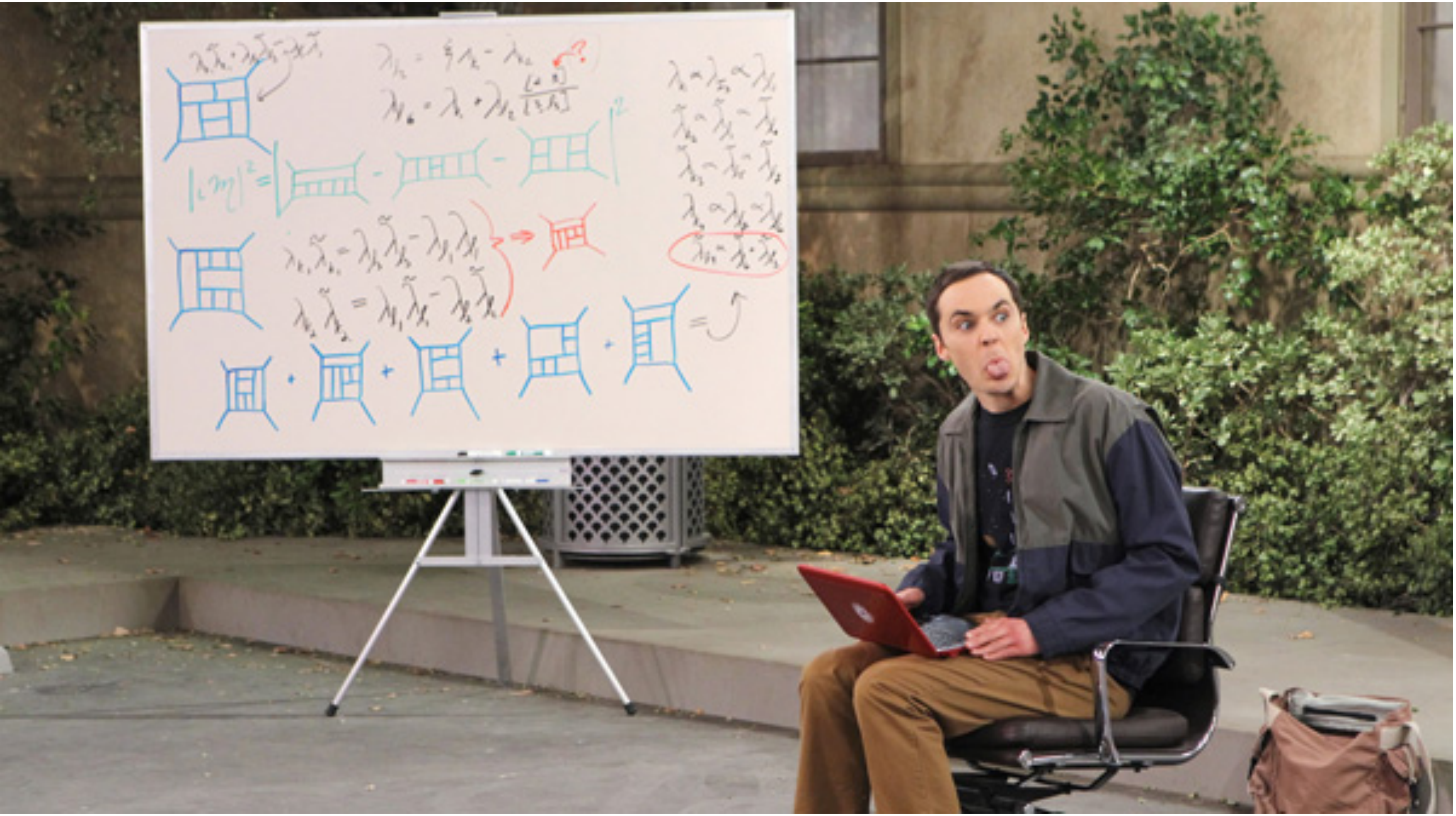
John Joseph shaved!

UV finite for $D = 4$ and 5

actually finite for $D < 11/2$

Very very finite.

Scene from “The Big Bang Theory”



A New Consensus from Experts

More recent papers argue that trouble starts at 5 loops and by 7 loops we have valid potential UV divergence in $D = 4$, accounting for known symmetries.

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Kallosh; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

All previous calculations explained and divergences predicted.

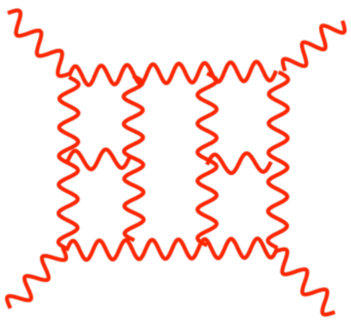
Based on this a reasonable person would conclude that $N = 8$ supergravity almost certainly diverges at 7 loops in $D = 4$.

Same consensus methods also predict:

- **$N = 8$ sugra should diverge at 7 loops in $D = 4$**
- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$**
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$**
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$**

$N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Johannson, Roiban

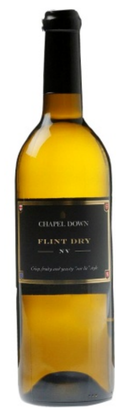


~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same.

Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**



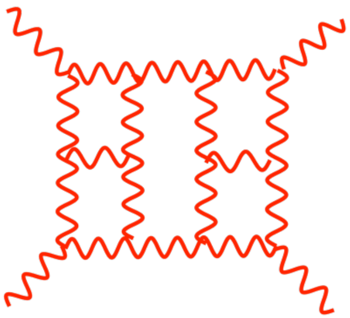
5 loops

Kelly Stelle:
English wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Johannson, Roiban



~1000 such diagrams with ~10,000s terms each

Being reasonable and being right are not the same

Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

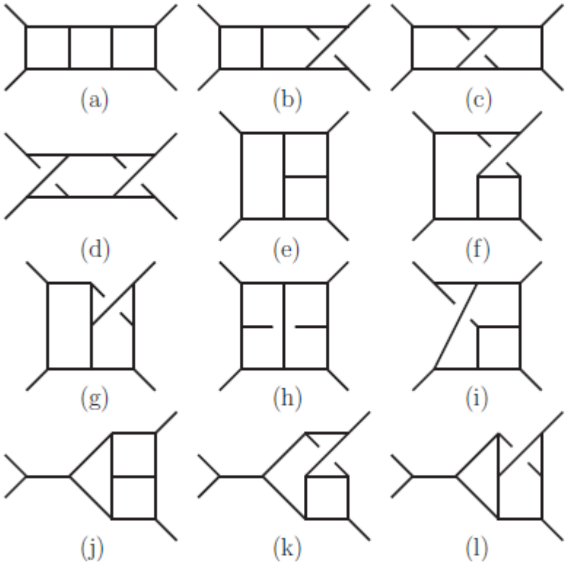
7 loops

David Gross:
California wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$\mathcal{N} = 4$ Supergravity UV Cancellation

ZB, Davies, Dennen, Huang



Graph	(divergence)/$(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

***All* three-loop divergences and subdivergences cancel completely!**

Still no symmetry explanation, despite valiant attempt.

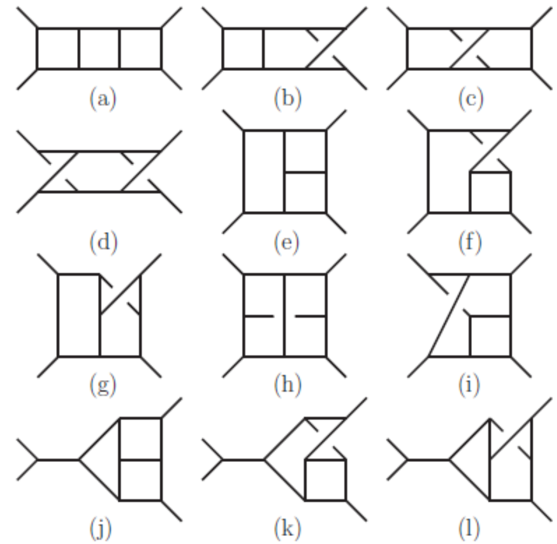
Bossard, Howe, Stelle; ZB, Davies, Dennen

A pity we did not bet on this theory!

Enhanced UV Cancellations

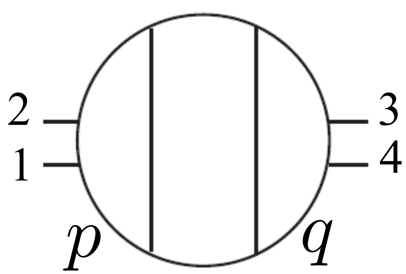
ZB, Davies, Dennen (2014)

Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.



- **By definition this is an enhanced cancellation.**
- **Not the way gauge theory works.**

***N* = 4 sugra**



already log divergent

***N* = 4 sugra: pure YM x *N* = 4 sYM**

$$n_i \sim s^3 t A_4^{\text{tree}} \, (p\cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

3 loop UV finiteness of *N* = 4 supergravity proves existence of "enhanced cancellation" in supergravity theories.

$\mathcal{N} = 5$ Supergravity at Four Loops

ZB, Davies and Dennen

We also calculated four-loop divergence in $\mathcal{N} = 5$ supergravity.

Industrial strength software needed: FIRE5 and C++

$\mathcal{N} = 5$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 1$ sYM)

Crucial help from FIRE5 and (Smirnov)^2

$\mathcal{N} = 4$ sYM **$\mathcal{N} = 1$ sYM**

Diagrams necessarily UV divergent.

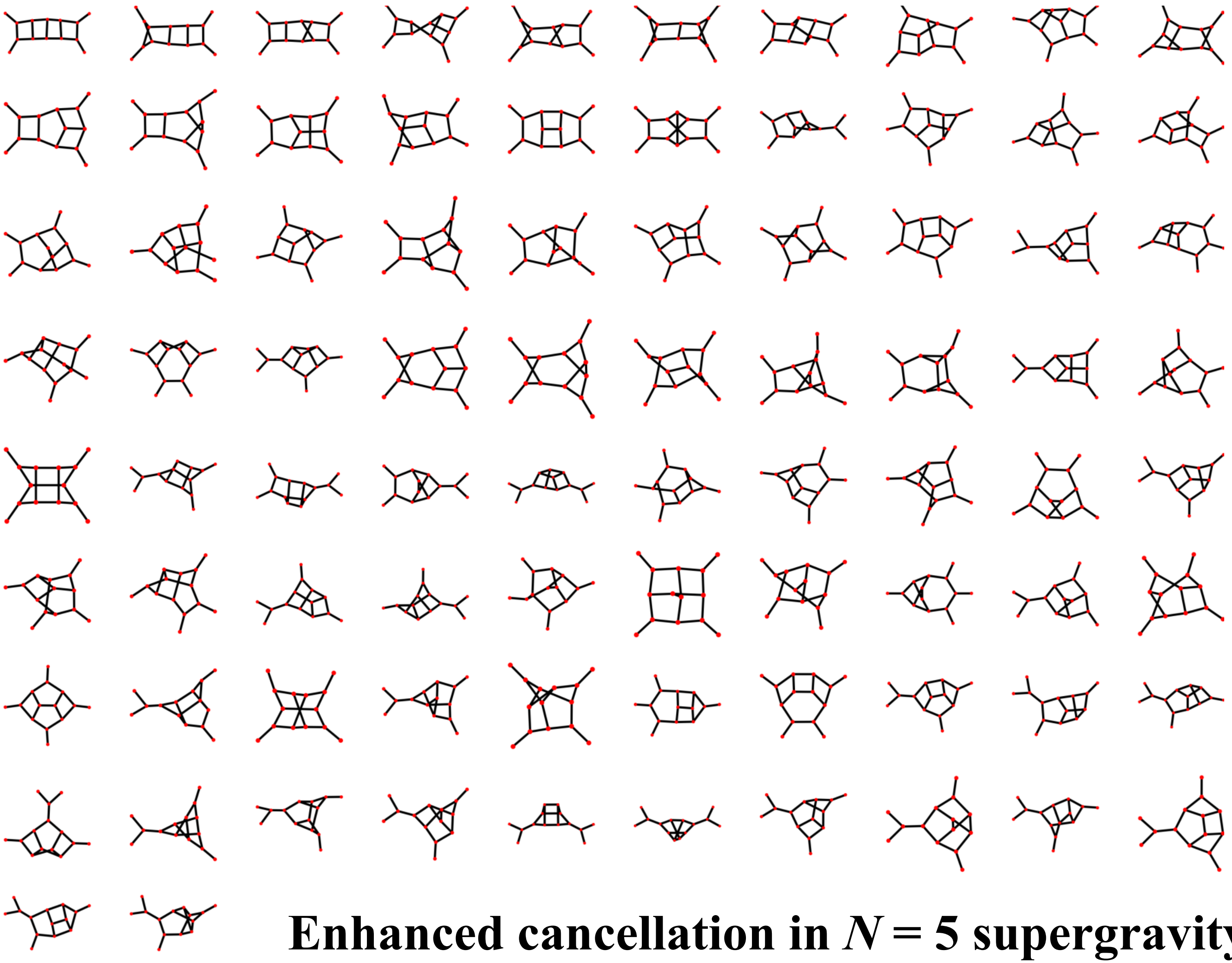
$\mathcal{N} = 5$ supergravity has no divergence at four loops.

Another example of an enhanced cancellation.

A pity we did not bet on this theory as well!

82 nonvanishing numerators in BCJ representation

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)



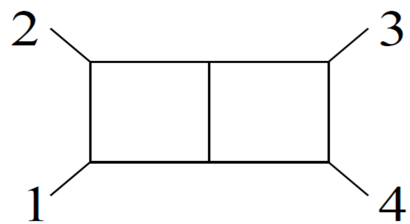
Enhanced cancellation in $\mathcal{N} = 5$ supergravity

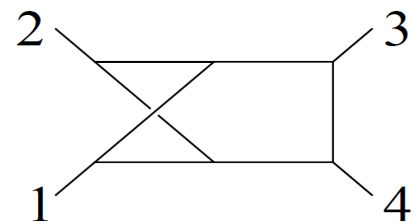
Where does new magic come from?

ZB, Davies, Dennen, Huang; Bossard, Howe, Stelle

To analyze we need a simpler example: Half-maximal supergravity in $D = 5$ at 2 loop.

Similar to $N = 4$, $D = 4$ sugra at 3 loops, except much simpler.





Quick summary:

- **— Finiteness in $D = 5$ tied to double-copy structure.**
- **— Cancellations in certain forbidden gauge-theory color structures imply hidden UV cancellations in supergravity, even when no standard symmetry explanation.**

Double copy structure implies extra cancellations!

Unfortunately, our 1, 2 loop proof not easy to extend beyond 2 loops.

The 4 loop Divergence of $N = 4$ Supergravity

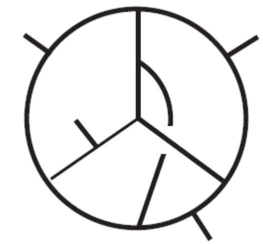
ZB, Davies, Dennen, A.V. Smirnov, V.A. Smirnov

At 4 loops $N = 4$ supergravity does have a UV divergence.
But structure is strange!

dim. reg. UV divergence

$$\mathcal{M}^{4\text{-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}$$

kinematic factor



$D = 4 - 2\epsilon$

It diverges but it has strange properties:

- **Contributions to helicity configurations that vanish were it not for a quantum anomaly. Vanishes classically.**
- **These helicity configuration have vanishing integrands in $D = 4$. Divergence is 0/0.**

Carrasco, Kallosh, Tseytlin and Roiban

Motivates closer examination of divergences.
Want simpler example: Pure Einstein gravity is simpler.

Two-Loop Pure Gravity

By two loops there is a valid R^3 counterterm and corresponding divergence.

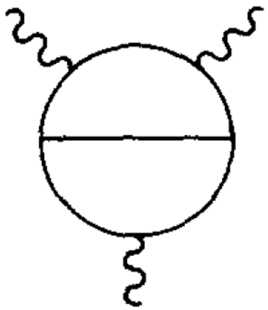
Goroff and Sagnotti (1986); Van de Ven (1992)

$$D = 4 - 2\epsilon$$

Divergence in pure Einstein gravity (no matter):

UV divergence

$$\mathcal{L}^{R^3} = \frac{209}{2880} \frac{1}{(4\pi)^4} \frac{1}{2\epsilon} R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}$$

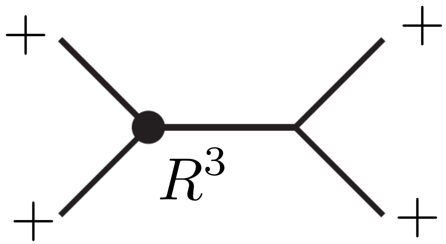


On surface nothing weird going on.

However, when we apply powerful modern tools we find results are subtle and weird, just like in $N = 4$ supergravity, once you probe carefully.

Two Loop Identical Helicity Amplitude

ZB, Cheung, Chi, Davies, Dixon and Nohle (2015)

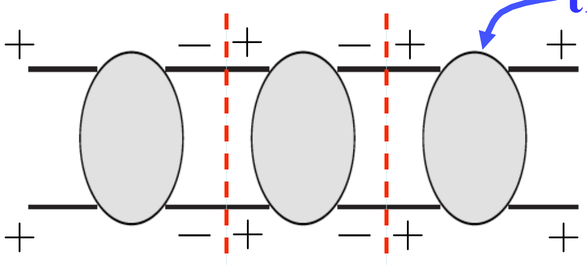


Pure gravity identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

$$\mathcal{M}^{R^3}\Big|_{\text{div.}} = \frac{209}{24\epsilon}\,\mathcal{K} \qquad \mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

Curious feature:

tree amplitude vanishes



- **Integrand naively vanishes!**
- **Nonvanishing because of 0/0 effect.**
- **Sign of quantum anomaly.**

Bardeen and Cangemi pointed out nonvanishing of identical helicity is connected to a quantum anomaly.

Surprises:

- **Divergence appears tied to anomalous behavior.**
- **Divergences don't appear meaningful. Divergences depend on nondynamical fields. Focus instead on renormalization scale.**

New Directions

If you want to solve a difficult problem get an army of energetic young people to help with new ideas:

- **Better understanding and applications of BCJ duality.**

Chiodaroli, Gunaydin, Johansson and Roiban,; Johannsson, Ochirov; O'Connell, Montiero, White; ZB, Davies, Nohle; Boels, Isermann, Monteiro, and O'Connel; Mogull and O'Connell, He, Monteiro, and Schlotterer

- **Scattering equations and double-copy relations.**

Cachazo, He, Yuan

- **Twistor strings now at loop level for $N = 8$ supergravity.**

Adamo, Casali and Skinner; Geyer, Mason, Monteiro and Tourkine

- **New ideas on unitarity cuts based on Feynman Tree Theorem**

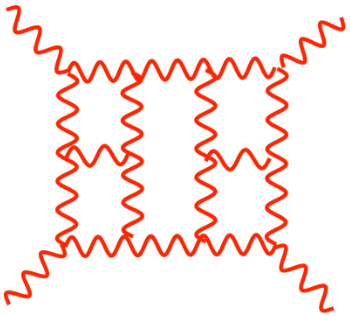
Baadsgaard, Bjerrum-Bohr, Bourjaily, Caron-Huot, Damgaard and Feng

- **Important advances in related string theory amplitudes.**

Carlos Mafra and Oliver Schlotterer

- **Awesome equation solver. Millions of equations encountered at 5 loops can be dealt with! Very cool algorithm!**

Schabinger and von Manteuffel



Outlook

- "Reports of the death of supergravity are an exaggeration"

  Stephan Hawking (with help from Mark Twain)

- UV finiteness of supergravity, given up for dead twice, is back in business, with new surprises: *Enhanced UV cancellations.*

- I don't know if this will lead to a completely satisfactory description of nature via supergravity. At least people are looking again at this possibility and we uncovered some interesting things along the way.

Summary

- Reformulated perturbative quantum field theory into a form allowing us to attack difficult fundamental problems. Is it possible to have perturbatively UV finite versions of Einstein gravity?

- Remarkable connection between gauge and gravity theories:
  — color $\longleftrightarrow$ kinematics.
  — gravity ~ (gauge theory)^2

- Pure supergravities surprisingly tame in the UV.
  New phenomenon: *Enhanced cancellations.*

- Strange anomaly-like behavior of divergences in gravity.
  Strange delinking of divergences from scaling behavior.

Supersymmetric versions of Einstein's General Relativity are surprisingly tame in the ultraviolet. Expect more surprises as we use modern tools to probe these theories.

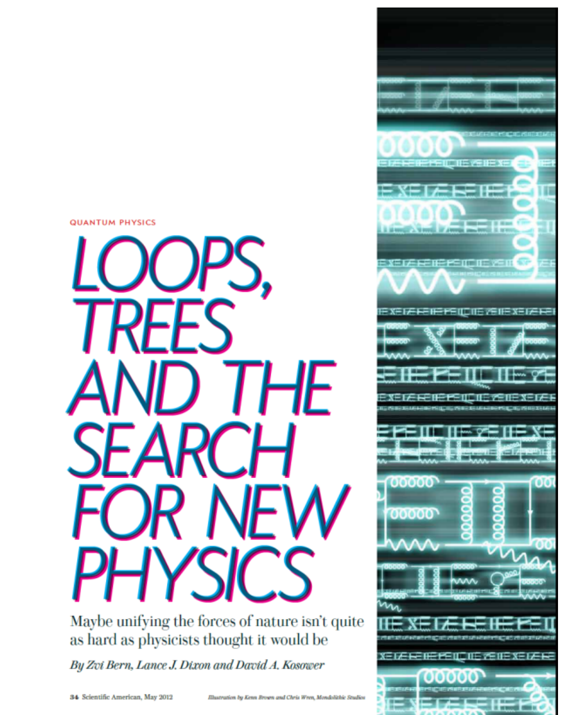
Further Reading

If you wish to read more see following non-technical descriptions.

Hermann Nicolai, *PRL Physics Viewpoint*, “Vanquishing Infinity”
http://physics.aps.org/articles/v2/70

Z. Bern, L. Dixon, D. Kosower,
May 2012 *Scientific American*,
“Loops, Trees and the Search for New Physics”

Anthony Zee, *Quantum Field Theory in a Nutshell*,
2nd Edition is first textbook to contain modern formulation of scattering and commentary on new developments. 4 new chapters.


QUANTUM PHYSICS

LOOPS, TREES AND THE SEARCH FOR NEW PHYSICS

Maybe unifying the forces of nature isn't quite as hard as physicists thought it would be

By Zvi Bern, Lance J. Dixon and David A. Kosower

34 Scientific American, May 2012

Illustration by Kenn Brown and Chris Wren, Mondolithic Studios